Serbia National Math Olympiad 2017

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–	Day 1	
1	Prove that for positive real numbers a, b, c such that $a + b + c = 1$, $$a\sqrt{2b+1} + b\sqrt{2c+1} + c\sqrt{2a+1} \leq \sqrt{2-(a^2+b^2+c^2)}.$$	
2	Let $ABCD$ be a convex and cyclic quadrilateral. Let $AD \cap BC = \{E\}$, and let M, N be points on AD, BC such that $AM : MD = BN : NC$. Circle around $\triangle EMN$ intersects circle around $ABCD$ at X, Y prove that AB, CD and XY are either parallel or concurrent.	
3	There are $2n-1$ lamps in a row. In the beginning only the middle one is on (the n-th one), and the other ones are off. Allowed move is to take two non-adjacent lamps which are turned off such that all lamps between them are turned on and switch all of their states from on to off and vice versa. What is the maximal number of moves until the process terminates?	
–	Day 2	
1	Let a be a positive integer.Suppose that $\forall n$,$\exists d$, $d \neq 1$, $d \equiv 1 \pmod{n}$,$d \mid n^2a - 1$.Prove that a is a perfect square.	
2	Find the maximum number of queens you could put on 2017×2017 chess table such that each queen attacks at most 1 other queen.	
3	Let k be the circumcircle of $\triangle ABC$ and let k_a be A-excircle .Let the two common tangents of k, k_a cut BC in P, Q.Prove that $\measuredangle PAB = \measuredangle CAQ$.	