ELMO Problems 1999

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1 In nonisosceles triangle ABC the excenters of the triangle opposite B and C be X_B and X_C, respectively. Let the external angle bisector of A intersect the circumcircle of $\triangle ABC$ again at Q. Prove that $QX_B = QB = QC = QX_C$.

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2 Mr. Fat moves around on the lattice points according to the following rules: From point (x, y) he may move to any of the points (y, x), $(3x, -2y)$, $(-2x, 3y)$, $(x + 1, y + 4)$ and $(x - 1, y - 4)$. Show that if he starts at $(0, 1)$ he can never get to $(0, 0)$.

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3 Prove that

$$2^6 \frac{abcd + 1}{(a + b + c + d)^2} \le a^2 + b^2 + c^2 + d^2 + \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} + \frac{1}{d^2}$$

for $a, b, c, d > 0$.

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4 Let $a_1, a_2, a_3, \cdots$ be an infinite sequence of real numbers. Prove that there exists a increasing sequence $j_1, j_2, j_3, \cdots$ of positive integers such that the sequence $a_{j_1}, a_{j_2}, a_{j_3}, \cdots$ is either nondecreasing or nonincreasing.
