ELMO Problems 2003

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by v_Enhance

1   Let $ABCDEF$ be a convex equilateral hexagon with sides of length 1. Let R_1 be the area of the region contained within both ACE and BDF, and let R_2 be the area of the region within the hexagon outside both triangles. Prove that:

$$\min\{[ACE], [BDF]\} + R_2 - R_1 \le \frac{3\sqrt{3}}{4}.$$

2   In a set of 30 MOPpers, prove that some two MOPpers have an even number of common friends.

3   Let k be a positive integer for which the equation

$$2ab + 2bc + 2ca - a^2 - b^2 - c^2 = k$$

has some solution in positive integers a, b, c. Prove that the equation has a solution for which a, b and c are the sides of a possibly degenerate triangle.

4   Let $x, y, z \ge 1$ be real numbers such that

$$\frac{1}{x^2 - 1} + \frac{1}{y^2 - 1} + \frac{1}{z^2 - 1} = 1.$$

Prove that

$$\frac{1}{x + 1} + \frac{1}{y + 1} + \frac{1}{z + 1} \le 1.$$