Nordic 2016

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by j___d

1   Determine all sequences of non-negative integers $a_1, \ldots, a_{2016}$ all less than or equal to 2016 satisfying $i + j \mid ia_i + ja_j$ for all $i, j \in \{1, 2, \ldots, 2016\}$.

2   Let $ABCD$ be a cyclic quadrilateral satysfing $AB = AD$ and $AB + BC = CD$. Determine $\angle CDA$.

3   Find all $a \in \mathbb{R}$ for which there exists a function $f \colon \mathbb{R} \to \mathbb{R}$, such that
(i) $f(f(x)) = f(x) + x$, for all $x \in \mathbb{R}$,
(ii) $f(f(x) - x) = f(x) + ax$, for all $x \in \mathbb{R}$.

4   King George has decided to connect the 1680 islands in his kingdom by bridges. Unfortunately the rebel movement will destroy two bridges after all the bridges have been built, but not two bridges from the same island. What is the minimal number of bridges the King has to build in order to make sure that it is still possible to travel by bridges between any two of the 1680 islands after the rebel movement has destroyed two bridges?