Paraguay Mathematical Olympiad 2007

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1 A list with 2007 positive integers is written on a board, such that the arithmetic mean of all the numbers is 12. Then, seven consecutive numbers are erased from the board. The arithmetic mean of the remaining numbers is 11.915.
The seven erased numbers have this property: the sixth number is half of the seventh, the fifth number is half of the sixth, and so on. Determine the 7 erased numbers.

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2 Let $ABCD$ be a square, such that the length of its sides are integers. This square is divided in 89 smaller squares, 88 squares that have sides with length 1, and 1 square that has sides with length n, where n is an integer larger than 1. Find all possible lengths for the sides of $ABCD$.

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3 Let $ABCD$ be a square, E and F midpoints of AB and AD respectively, and P the intersection of CF and DE.
a) Show that $DE \perp CF$.
b) Determine the ratio $CF : PC : EP$

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4 Each number from the set $\{1, 2, 3, 4, 5, 6, 7\}$ must be written in each circle of the diagram, so that the sum of any three *aligned* numbers is the same (e.g., $A + D + E = D + C + B$). What number cannot be placed on the circle E?

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5 Let $A, B, C,$ be points in the plane, such that we can draw 3 equal circumferences in which the first one passes through A and B, the second one passes through B and C, the last one passes through C and A, and all 3 circumferences share a common point P.
Show that the radius of each of these circumferences is equal to the circumradius of triangle ABC, and that P is the orthocenter of triangle ABC.
