Paraguay Mathematical Olympiad 2008

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by Leicich

1 How many positive integers $n < 500$ exist such that its prime factors are exclusively 2, 7, 11, or a combination of these?

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2 Find for which values of n, an integer larger than 1 but smaller than 100, the following expression has its minimum value:

$S = |n-1| + |n-2| + \ldots + |n-100|$

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3 Let ABC be a triangle, where $AB = AC$ and $BC = 12$. Let D be the midpoint of BC. Let E be a point in AC such that $DE \perp AC$. Let F be a point in AB such that $EF \parallel BC$. If $EC = 4$, determine the length of EF.

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4 Let Γ be a circumference and A a point outside it. Let B and C be points in Γ such that AB and AC are tangent to Γ. Let P be a point in Γ. Let D, E and F be points in BC, AC and AB respectively, such that $PD \perp BC$, $PE \perp AC$, and $PF \perp AB$.
Show that $PD^2 = PE \cdot PF$

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5 Let m, n, p be rational numbers such that $\sqrt{m} + \sqrt{n} + \sqrt{p}$ is a rational number. Prove that $\sqrt{m}, \sqrt{n}, \sqrt{p}$ are also rational numbers

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