

**Paraguay Mathematical Olympiad 2009**

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by Leichich

- 1 Find the value of the following expression:

$$2 + 33 + 6 + 35 + 10 + 37 + \dots + 1194 + 629 + 1198 + 631$$

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- 2 In a triangle  $ABC$  ( $\angle BCA = 90^\circ$ ), let  $D$  be the intersection of  $AB$  with a circumference with diameter  $BC$ . Let  $F$  be the intersection of  $AC$  with a line tangent to the circumference. If  $\angle CAB = 46^\circ$ , find the measure of  $\angle CFD$ .
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- 3 Find out how many positive integers  $n$  not larger than 2009 exist such that the last digit of  $n^{20}$  is 1.
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- 4 Let  $a_1, a_2, \dots, a_n$  be a sequence such that the arithmetic mean of the  $n$  terms is  $n$ . Consider  $n = 2009$ . Determine the sum of the 2009 terms of the sequence.
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- 5 In a triangle  $ABC$ , let  $I$  be its incenter. The distance from  $I$  to the segment  $BC$  is  $4\text{cm}$  and the distance from that point to vertex  $B$  is  $12\text{cm}$ . Let  $D$  be a point in the plane region between segments  $AB$  and  $BC$  such that  $D$  is the center of a circumference that is tangent to lines  $AB$  and  $BC$  and passes through  $I$ . Find all possible values of the length  $BD$ .
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