Paraguay Mathematical Olympiad 2010

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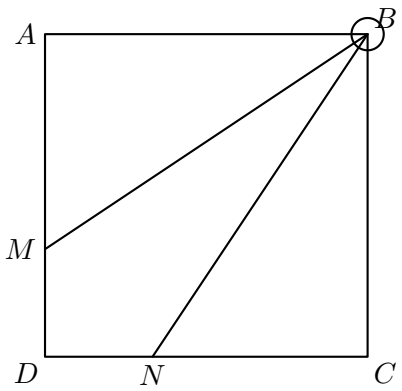
1 The picture below shows the way Juan wants to divide a square field in three regions, so that all three of them share a well at vertex B. If the side length of the field is 60 meters, and each one of the three regions has the same area, how far must the points M and N be from D?

Note: the area of each region includes the area the well occupies.



2 A series of figures is shown in the picture below, each one of them created by following a secret rule. If the leftmost figure is considered the first figure, how many squares will the 21st figure have?

http://www.artofproblemsolving.com/Forum/download/file.php?id=49934

Note: only the little squares are to be counted (i.e., the 2×2 squares, 3×3 squares, ... should not be counted)

Extra (not part of the original problem): How many squares will the 21st figure have, if we consider all 1×1 squares, all 2×2 squares, all 3×3 squares, and so on?.

3 In a triangle ABC, let M be the midpoint of AC. If $BC = \frac{2}{3}MC$ and $\angle BMC = 2\angle ABM$, determine $\frac{AM}{AB}$.

4 Find all 4-digit numbers $\overline{abcd}$ that are multiples of 11, such that the 2-digit number $\overline{ac}$ is a

multiple of 7 and $a+b+c+d=d^2$.

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5 In a triangle ABC, let D, E and F be the feet of the altitudes from A, B and C respectively. Let D', E' and F' be the second intersection of lines AD, BE and CF with the circumcircle of ABC. Show that the triangles DEF and $D'E'F'$ are similar.
