Paraguay Mathematical Olympiad 2011

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by Leicich

1   Find the value of the following expression:

$\frac{1}{2} + (\frac{1}{3} + \frac{2}{3}) + (\frac{1}{4} + \frac{2}{4} + \frac{3}{4}) + \ldots + (\frac{1}{1000} + \frac{2}{1000} + \ldots + \frac{999}{1000})$

2   In a triangle ABC, let D and E be the midpoints of AC and BC respectively. The distance from the midpoint of BD to the midpoint of AE is 4.5. What is the length of side AB?

3   If number $\overline{aaaa}$ is divided by $\overline{bb}$, the quotient is a number between 140 and 160 inclusively, and the remainder is equal to $\overline{(a-b)(a-b)}$. Find all pairs of positive integers (a, b) that satisfy this.

4   A positive integer N is divided in n parts inversely proportional to the numbers $2, 6, 12, 20, \ldots$ The smallest part is equal to $\frac{1}{400}N$. Find the value of n.

5   In a rectangle triangle, let I be its incenter and G its geocenter. If IG is parallel to one of the catheti and measures $10cm$, find the lengths of the two catheti of the triangle.