Paraguay Mathematical Olympiad 2012

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1	Define a list of number with the following properties: - The first number of the list is a one-digit natural number. - Each number (since the second) is obtained by adding 9 to the number before in the list. - The number 2012 is in that list. Find the first number of the list.
2	The *traveler ant* is walking over several chess boards. He only walks vertically and horizontally through the squares of the boards and does not pass two or more times over the same square of a board. a) In a 4x4 board, from which squares can he begin his travel so that he can pass through all the squares of the board? b) In a 5x5 board, from which squares can he begin his travel so that he can pass through all the squares of the board? c) In a nxn board, from which squares can he begin his travel so that he can pass through all the squares of the board?
3	Let ABC be a triangle (right in B) inscribed in a semi-circumference of diameter $AC = 10$. Determine the distance of the vertice B to the side AC if the median corresponding to the hypotenuse is the geometric mean of the sides of the triangle.
4	Find all four-digit numbers $\overline{abcd}$ such that they are multiples of 3 and that $\overline{ab} - \overline{cd} = 11$. ($\overline{abcd}$ is a four-digit number; $\overline{ab}$ is a two digit-number as $\overline{cd}$ is).
5	Let ABC be an equilateral triangle. Let Q be a random point on BC, and let P be the meeting point of AQ and the circumscribed circle of $\triangle ABC$. Prove that $\frac{1}{PQ} = \frac{1}{PB} + \frac{1}{PC}$.