**F = Ma 2007**

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**1** An object moves in two dimensions according to

$$\vec{r}(t) = (4.0t^2 - 9.0)\vec{i} + (2.0t - 5.0)\vec{j}$$

where $r$ is in meters and $t$ in seconds. When does the object cross the $x$-axis?

**(A)** $0.0$ s **(B)** $0.4$ s **(C)** $0.6$ s **(D)** $1.5$ s **(E)** $2.5$ s

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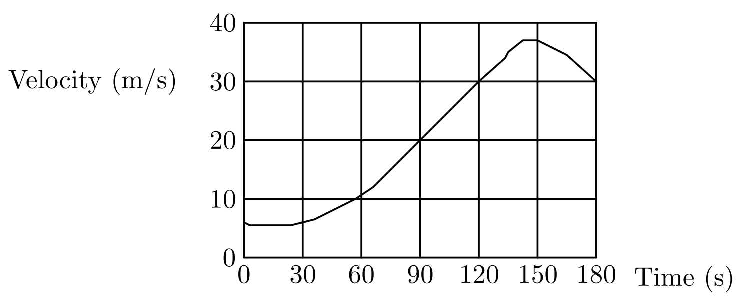
**2** The graph shows velocity as a function of time for a car. What was the acceleration at time = $90$ seconds?




**(A)** $0.2$ m/s$^2$ **(B)** $0.33$ m/s$^2$ **(C)** $1.0$ m/s$^2$ **(D)** $9.8$ m/s$^2$ **(E)** $30$ m/s$^2$

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**3** The coordinate of an object is given as a function of time by $x = 8t - 3t^2$, where $x$ is in meters and $t$ is in seconds. Its average velocity over the interval from $t = 1$ to $t = 2$ s is

**(A)** $-2$ m/s **(B)** $-1$ m/s **(C)** $-0.5$ m/s **(D)** $0.5$ m/s **(E)** $1$ m/s

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**4** An object is released from rest and falls a distance $h$ during the first second of time. How far will it fall during the next second of time?

**(A)** $h$ **(B)** $2h$ **(C)** $3h$ **(D)** $4h$ **(E)** $h^2$

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**5** A crate of toys remains at rest on a sleigh as the sleigh is pulled up a hill with an increasing speed. The crate is not fastened down to the sleigh. What force is responsible for the crates increase in speed up the hill?

**(A)** the force of static friction of the sleigh on the crate

**(B)** the contact force (normal force) of the ground on the sleigh

**(C)** the contact force (normal force) of the sleigh on the crate

**(D)** the gravitational force acting on the sleigh

**(E)** no force is needed

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**6** At time $t = 0$ a drag racer starts from rest at the origin and moves along a straight line with velocity given by $v = 5t^2$, where $v$ is in m/s and $t$ in s. The expression for the displacement of the car from $t = 0$ to time $t$ is

**(A)** $5t^3$ **(B)** $5t^3/3$ **(C)** $10t$ **(D)** $15t^2$ **(E)** $5t/2$

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**7** The chemical potential energy stored in a battery is converted into kinetic energy in a toy car that increases its speed first from $0$ mph to $2$ mph and then from $2$ mph up to $4$ mph. Ignore the energy transferred to thermal energy due to friction and air resistance. Compared to the energy required to go from $0$ to $2$ mph, the energy required to go from $2$ to $4$ mph is

**(A)** half the amount.

**(B)** the same amount.

**(C)** twice the amount.

**(D)** three times the amount.

**(E)** four times the amount.

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**8** When two stars are very far apart their gravitational potential energy is zero; when they are separated by a distance $d$ the gravitational potential energy of the system is $U$. If the stars are separated by a distance $2d$ the gravitational potential energy of the system is

**(A)** $U/4$ **(B)** $U/2$ **(C)** $U$ **(D)** $2U$ **(E)** $4U$

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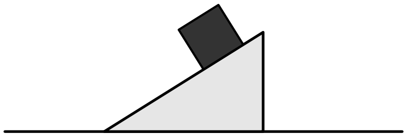
**9** A large wedge rests on a horizontal frictionless surface, as shown. A block starts from rest and slides down the inclined surface of the wedge, which is rough. During the motion of the block, the center of mass of the block and wedge



**(A)** does not move

**(B)** moves horizontally with constant speed

**(C)** moves horizontally with increasing speed

**(D)** moves vertically with increasing speed

**(E)** moves both horizontally and vertically

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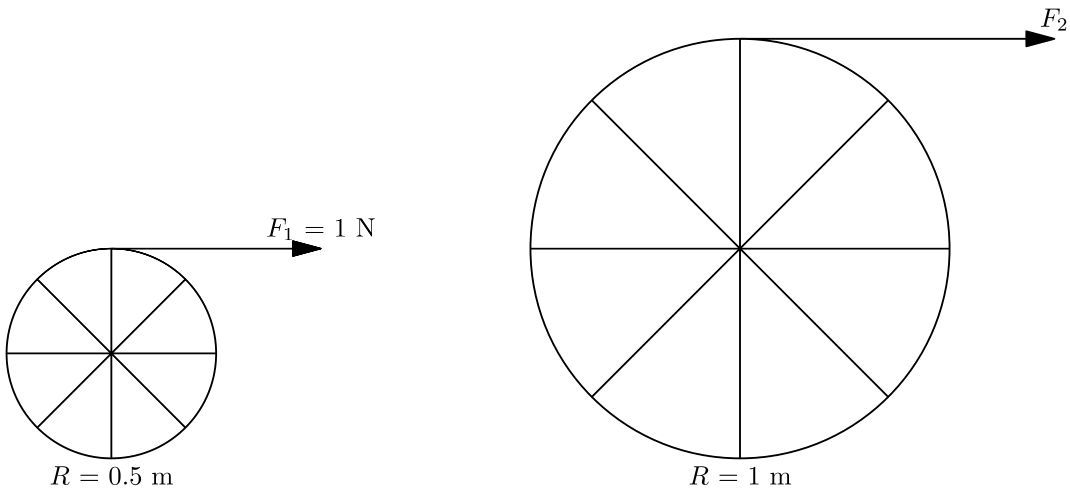
**10** Two wheels with fixed hubs, each having a mass of $1$ kg, start from rest, and forces are applied as shown. Assume the hubs and spokes are massless, so that the rotational inertia is $I = mR^2$. In order to impart identical angular accelerations about their respective hubs, how large must $F_2$ be?




**(A)** $0.25$ N **(B)** $0.5$ N **(C)** $1$ N **(D)** $2$ N **(E)** $4$ N

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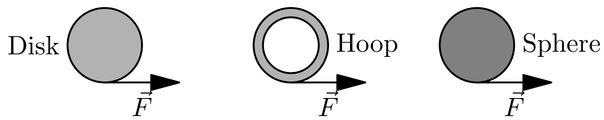
**11** A uniform disk, a thin hoop, and a uniform sphere, all with the same mass and same outer radius, are each free to rotate about a fixed axis through its center. Assume the hoop is connected to the rotation axis by light spokes. With the objects starting from rest, identical forces are simultaneously applied to the rims, as shown. Rank the objects according to their kinetic energies after a given time $t$, from least to greatest.




**(A)** disk, hoop, sphere

**(B)** sphere, disk, hoop

**(C)** hoop, sphere, disk

**(D)** disk, sphere, hoop

**(E)** hoop, disk, sphere

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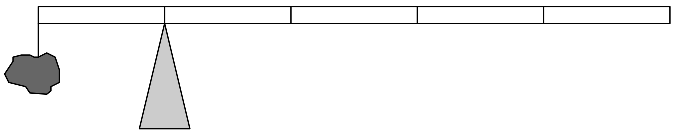
**12** A $2$-kg rock is suspended by a massless string from one end of a uniform $1$-meter measuring stick. What is the mass of the measuring stick if it is balanced by a support force at the $0.20$-meter mark?



**(A)** $0.20$ kg **(B)** $1.00$ kg **(C)** $1.33$ kg **(D)** $2.00$ kg **(E)** $3.00$ kg

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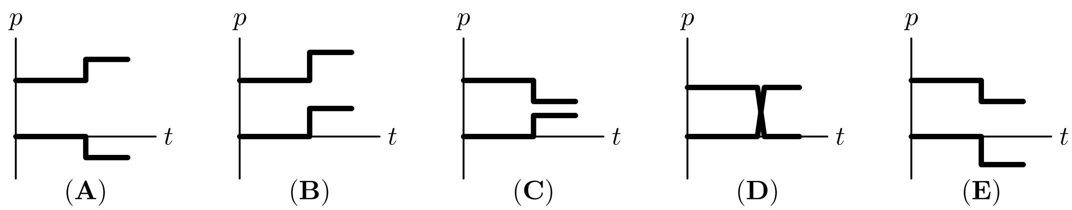
**13** A particle moves along the $x$-axis. It collides elastically head-on with an identical particle initially at rest. Which of the following graphs could illustrate the momentum of each particle as a function of time?

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**14** When the speed of a rear-drive car is increasing on a horizontal road, the direction of the frictional force on the tires is:

**(A)** backward on the front tires and forward on the rear tires.

**(B)** forward on the front tires and backward on the rear tires.

**(C)** forward on all tires.

**(D)** backward on all tires.

**(E)** zero.

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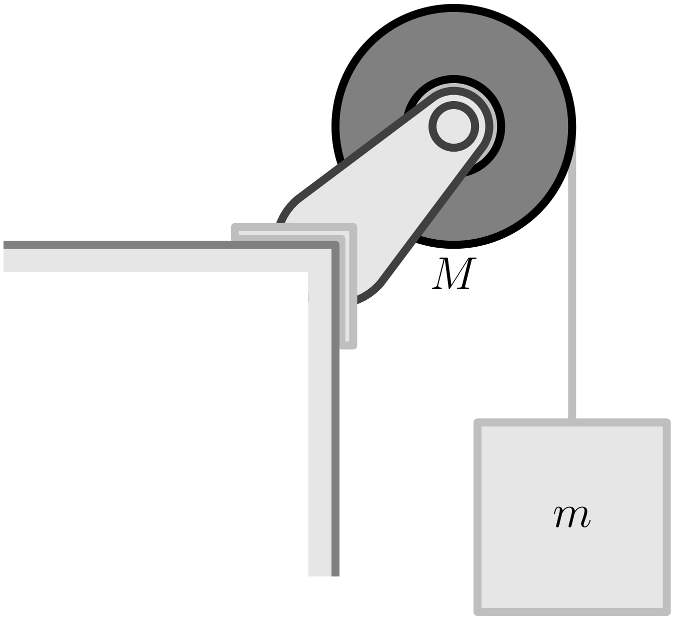
**15** A uniform disk ($I = \frac{1}{2}MR^2$) of mass 8.0 kg can rotate without friction on a fixed axis. A string is wrapped around its circumference and is attached to a 6.0 kg mass. The string does not slip. What is the tension in the cord while the mass is falling?




**(A)** 20.0 N **(B)** 24.0 N **(C)** 34.3 N **(D)** 60.0 N **(E)** 80.0 N

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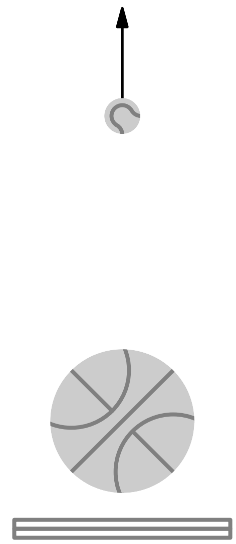
**16** A baseball is dropped on top of a basketball. The basketball hits the ground, rebounds with a speed of $4.0$ m/s, and collides with the baseball as it is moving downward at $4.0$ m/s. After the collision, the baseball moves upward as shown in the figure and the basketball is instantaneously at rest right after the collision. The mass of the baseball is $0.2$ kg and the mass of the basketball is $0.5$ kg. Ignore air resistance and ignore any changes in velocities due to gravity during the very short collision times. The speed of the baseball right after colliding with the upward moving basketball is



**(A)** $4.0$ m/s **(B)** $6.0$ m/s **(C)** $8.0$ m/s **(D)** $12.0$ m/s **(E)** $16.0$ m/s

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**17** A small point-like object is thrown horizontally off of a $50.0$-m high building with an initial speed of $10.0$ m/s. At any point along the trajectory there is an acceleration component tangential to the trajectory and an acceleration component perpendicular to the trajectory. How many seconds after the object is thrown is the tangential component of the acceleration of the object equal to twice the perpendicular component of the acceleration of the object? Ignore air resistance.

**(A)** $2.00$ s

**(B)** $1.50$ s

**(C)** $1.00$ s

**(D)** $0.50$ s

**(E)** The building is not high enough for this to occur.

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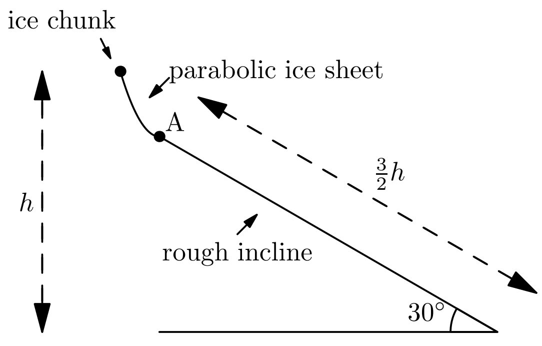
**18** A small chunk of ice falls from rest down a frictionless parabolic ice sheet shown in the figure. At the point labeled $\mathbf{A}$ in the diagram, the ice sheet becomes a steady, rough incline of angle $30^\circ$ with respect to the horizontal and friction coefficient $\mu_k$. This incline is of length $\frac{3}{2}h$ and ends at a cliff. The chunk of ice comes to a rest precisely at the end of the incline. What is the coefficient of friction $\mu_k$?




**(A)** $0.866$ **(B)** $0.770$ **(C)** $0.667$ **(D)** $0.385$ **(E)** $0.333$

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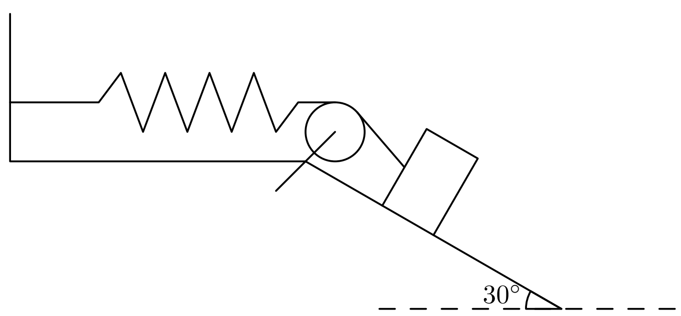
**19** A non-Hookian spring has force $F = -kx^2$ where $k$ is the spring constant and $x$ is the displacement from its unstretched position. For the system shown of a mass $m$ connected to an unstretched spring initially at rest, how far does the spring extend before the system momentarily comes to rest? Assume that all surfaces are frictionless and that the pulley is frictionless as well.




**(A)** $\left(\frac{3mg}{2k}\right)^{1/2}$

**(B)** $\left(\frac{mg}{k}\right)^{1/2}$

**(C)** $\left(\frac{2mg}{k}\right)^{1/2}$

**(D)** $\left(\frac{\sqrt{3}mg}{k}\right)^{1/3}$

**(E)** $\left(\frac{3\sqrt{3}mg}{2k}\right)^{1/3}$

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**20** A point-like mass moves horizontally between two walls on a frictionless surface with initial kinetic energy $E$. With every collision with the walls, the mass loses $1/2$ its kinetic energy to thermal energy. How many collisions with the walls are necessary before the speed of the mass is reduced by a factor of $8$?

**(A)** $3$ **(B)** $4$ **(C)** $6$ **(D)** $8$ **(E)** $16$

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**21** If the rotational inertia of a sphere about an axis through the center of the sphere is $I$, what is the rotational inertia of another sphere that has the same density, but has twice the radius?

**(A)** $2I$ **(B)** $4I$ **(C)** $8I$ **(D)** $16I$ **(E)** $32I$

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**22** Two rockets are in space in a negligible gravitational field. All observations are made by an observer in a reference frame in which both rockets are initially at rest. The masses of the rockets are $m$ and $9m$. A constant force $F$ acts on the rocket of mass m for a distance $d$. As a result, the rocket acquires a momentum $p$. If the same constant force $F$ acts on the rocket of mass $9m$ for the same distance $d$, how much momentum does the rocket of mass $9m$ acquire?

**(A)** $p/9$ **(B)** $p/3$ **(C)** $p$ **(D)** $3p$ **(E)** $9p$

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**23** If a planet of radius $R$ spins with an angular velocity $\omega$ about an axis through the North Pole, what is the ratio of the normal force experienced by a person at the equator to that experienced by a person at the North Pole? Assume a constant gravitational field $g$ and that both people are stationary relative to the planet and are at sea level.

**(A)** $g/R\omega^2$

**(B)** $R\omega^2/g$

**(C)** $1 - R\omega^2/g$

**(D)** $1 + g/R\omega^2$

**(E)** $1 + R\omega^2/g$

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**24** A ball of mass $m$ is launched into the air. Ignore air resistance, but assume that there is a wind that exerts a constant force $F_0$ in the -$x$ direction. In terms of $F_0$ and the acceleration due to gravity $g$, at what angle above the positive $x$-axis must the ball be launched in order to come back to the point from which it was launched?

**(A)** $\tan^{-1}(F_0/mg)$

**(B)** $\tan^{-1}(mg/F_0)$

**(C)** $\sin^{-1}(F_0/mg)$

**(D)** the angle depends on the launch speed

**(E)** no such angle is possible

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**25** Find the period of small oscillations of a water pogo, which is a stick of mass m in the shape of a box (a rectangular parallelepiped.) The stick has a length $L$, a width $w$ and a height $h$ and is bobbing up and down in water of density $\rho$ . Assume that the water pogo is oriented such that the length $L$ and width $w$ are horizontal at all times. Hint: The buoyant force on an object is given by $F_{buoy} = \rho V g$, where $V$ is the volume of the medium displaced by the object and $\rho$ is the density of the medium. Assume that at equilibrium, the pogo is floating.

**(A)** $2\pi\sqrt{\frac{L}{g}}$

**(B)** $\pi\sqrt{\frac{\rho w^2L^2g}{mh^2}}$

**(C)** $2\pi\sqrt{\frac{mh^2}{\rho L^2w^2g}}$

**(D)** $2\pi\sqrt{\frac{m}{\rho wLg}}$

**(E)** $\pi\sqrt{\frac{m}{\rho wLg}}$

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**26** A sled loaded with children starts from rest and slides down a snowy $25^\circ$ (with respect to the horizontal) incline traveling $85$ meters in $17$ seconds. Ignore air resistance. What is the coefficient of kinetic friction between the sled and the slope?

**(A)** $0.36$ **(B)** $0.40$ **(C)** $0.43$ **(D)** $1.00$ **(E)** $2.01$

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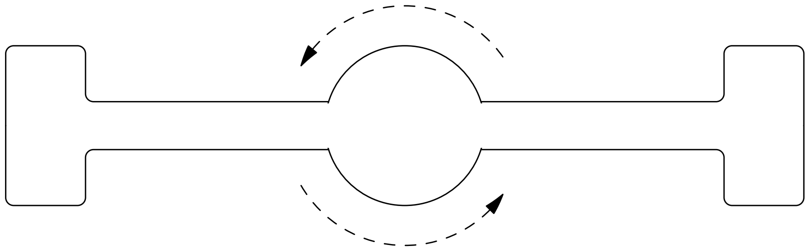
**27** A space station consists of two living modules attached to a central hub on opposite sides of the hub by long corridors of equal length. Each living module contains $N$ astronauts of equal mass. The mass of the space station is negligible compared to the mass of the astronauts, and the size of the central hub and living modules is negligible compared to the length of the corridors. At the beginning of the day, the space station is rotating so that the astronauts feel as if they are in a gravitational field of strength $g$. Two astronauts, one from each module, climb into the central hub, and the remaining astronauts now feel a gravitational field of strength $g'$ . What is the ratio $g'/g$ in terms of $N$?



**(A)** $2N/(N-1)$

**(B)** $N/(N-1)$

**(C)** $\sqrt{(N-1)/N}$

**(D)** $\sqrt{N/(N-1)}$

**(E)** none of the above

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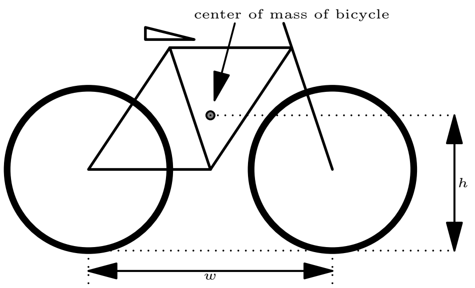
**28** A simplified model of a bicycle of mass $M$ has two tires that each comes into contact with the ground at a point. The wheelbase of this bicycle (the distance between the points of contact with the ground) is $w$, and the center of mass of the bicycle is located midway between the tires and a height h above the ground. The bicycle is moving to the right, but slowing down at a constant rate. The acceleration has a magnitude $a$. Air resistance may be ignored.




Case 1 (**Questions 28 - 29**): Assume that the coefficient of sliding friction between each tire and the ground is $\mu$, and that both tires are skidding: sliding without rotating. Express your answers in terms of $w$, $h$, $M$, and $g$.

What is the maximum value of $\mu$ so that both tires remain in contact with the ground?

**(A)** $\frac{w}{2h}$

**(B)** $\frac{h}{2w}$

**(C)** $\frac{2h}{w}$

**(D)** $\frac{w}{h}$

**(E)** none of the above

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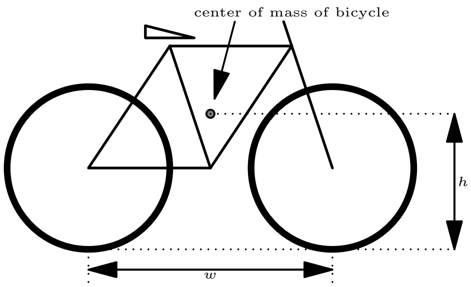
**29** A simplified model of a bicycle of mass $M$ has two tires that each comes into contact with the ground at a point. The wheelbase of this bicycle (the distance between the points of contact with the ground) is $w$, and the center of mass of the bicycle is located midway between the tires and a height h above the ground. The bicycle is moving to the right, but slowing down at a constant rate. The acceleration has a magnitude $a$. Air resistance may be ignored.




Case 1 (**Questions 28 - 29**): Assume that the coefficient of sliding friction between each tire and the ground is $\mu$, and that both tires are skidding: sliding without rotating. Express your answers in terms of $w$, $h$, $M$, and $g$.

What is the maximum value of $a$ so that both tires remain in contact with the ground?

**(A)** $\frac{wg}{h}$

**(B)** $\frac{wg}{2h}$

**(C)** $\frac{hg}{2w}$

**(D)** $\frac{h}{2wg}$

**(E)** none of the above

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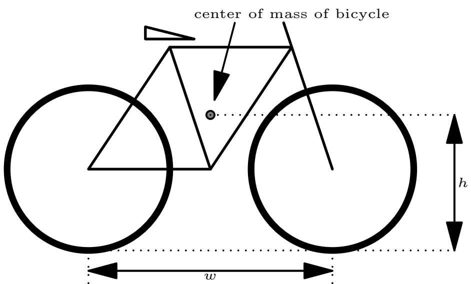
**30** A simplified model of a bicycle of mass $M$ has two tires that each comes into contact with the ground at a point. The wheelbase of this bicycle (the distance between the points of contact with the ground) is $w$, and the center of mass of the bicycle is located midway between the tires and a height h above the ground. The bicycle is moving to the right, but slowing down at a constant rate. The acceleration has a magnitude $a$. Air resistance may be ignored.




Case 2 (**Question 30**): Assume, instead, that the coefficient of sliding friction between each

tire and the ground is different: $\mu_1$ for the front tire and $\mu_2$ for the rear tire. Let $\mu_1 = 2\mu_2$.

Assume that both tires are skidding: sliding without rotating. What is the maximum value of $a$ so that both tires remain in contact with the ground?

**(A)** $\frac{wg}{h}$

**(B)** $\frac{wg}{3h}$

**(C)** $\frac{2wg}{3h}$

**(D)** $\frac{hg}{2w}$

**(E)** none of the above

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**31** A thin, uniform rod has mass $m$ and length $L$. Let the acceleration due to gravity be $g$. Let the rotational inertia of the rod about its center be $md^2$.

Find the ratio $L/d$.

**(A)** $3\sqrt{2}$ **(B)** $3$ **(C)** $12$ **(D)** $2\sqrt{3}$ **(E)** none of the above

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**32** A thin, uniform rod has mass $m$ and length $L$. Let the acceleration due to gravity be $g$. Let the rotational inertia of the rod about its center be $md^2$.

The rod is suspended from a distance $kd$ from the center, and undergoes small oscillations with an angular frequency $\beta\sqrt{\frac{g}{d}}$.

Find an expression for $\beta$ in terms of $k$.

**(A)** $1 + k^2$

**(B)** $\sqrt{1 + k^2}$

**(C)** $\sqrt{\frac{k}{1+k}}$

**(D)** $\sqrt{\frac{k^2}{1+k}}$

**(E)** none of the above

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**33** A thin, uniform rod has mass $m$ and length $L$. Let the acceleration due to gravity be $g$. Let the rotational inertia of the rod about its center be $md^2$.

The rod is suspended from a distance $kd$ from the center, and undergoes small oscillations with an angular frequency $\beta\sqrt{\frac{g}{d}}$.

Find the maximum value of $\beta$.

**(A)** $1$

**(B)** $\sqrt{2}$

**(C)** $1/\sqrt{2}$

**(D)** $\beta$ does not attain a maximum value

**(E)** none of the above

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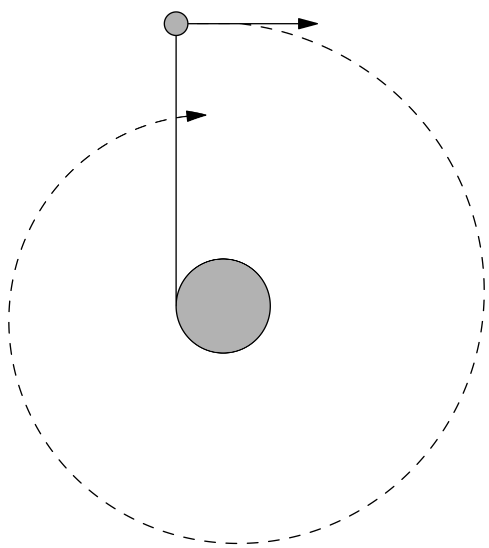
**34** A point object of mass $m$ is connected to a cylinder of radius $R$ via a massless rope. At time $t = 0$ the object is moving with an initial velocity $v_0$ perpendicular to the rope, the rope has a length $L_0$, and the rope has a non-zero tension. All motion occurs on a horizontal frictionless surface. The cylinder remains stationary on the surface and does not rotate. The object moves in such a way that the rope slowly winds up around the cylinder. The rope will break when the tension exceeds $T_{max}$. Express your answers in terms of $T_{max}$, $m$, $L_0$, $R$, and $v_0$.



What is the angular momentum of the object with respect to the axis of the cylinder at the instant that the rope breaks?

**(A)** $mv_0R$

**(B)** $\frac{m^2v_0^3}{T_{max}}$

**(C)** $mv_0L_0$

**(D)** $\frac{T_{max}R^2}{v_0}$

**(E)** none of the above

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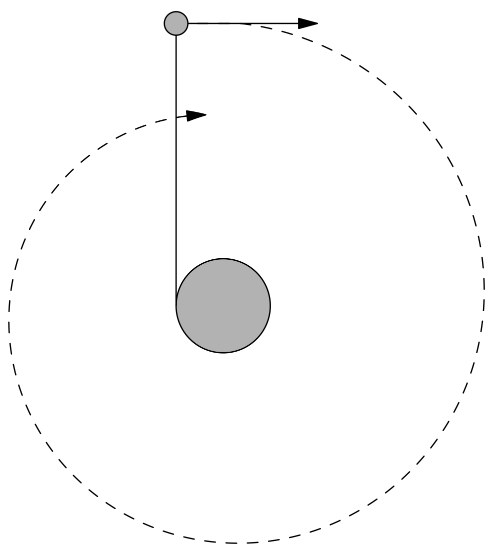
**35** A point object of mass $m$ is connected to a cylinder of radius $R$ via a massless rope. At time $t = 0$ the object is moving with an initial velocity $v_0$ perpendicular to the rope, the rope has a length $L_0$, and the rope has a non-zero tension. All motion occurs on a horizontal frictionless surface. The cylinder remains stationary on the surface and does not rotate. The object moves in such a way that the rope slowly winds up around the cylinder. The rope will break when the tension exceeds $T_{max}$. Express your answers in terms of $T_{max}$, $m$, $L_0$, $R$, and $v_0$.



What is the kinetic energy of the object at the instant that the rope breaks?

**(A)** $\frac{mv_0^2}{2}$

**(B)** $\frac{mv_0^2 R}{2L_0}$

**(C)** $\frac{mv_0^2 R^2}{2L_0^2}$

**(D)** $\frac{mv_0^2 L_0^2}{2R^2}$

**(E)** none of the above

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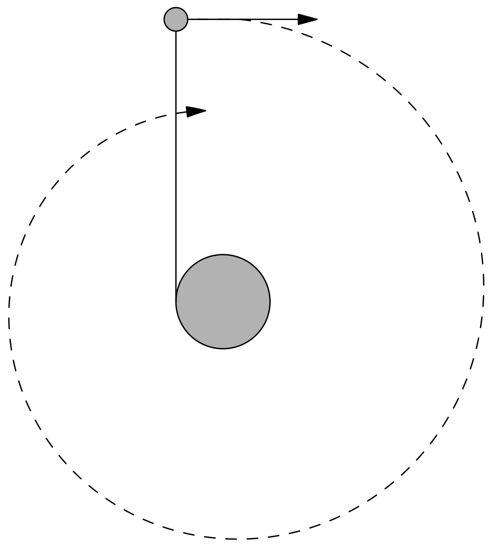
**36** A point object of mass $m$ is connected to a cylinder of radius $R$ via a massless rope. At time $t = 0$ the object is moving with an initial velocity $v_0$ perpendicular to the rope, the rope has a length $L_0$, and the rope has a non-zero tension. All motion occurs on a horizontal frictionless surface. The cylinder remains stationary on the surface and does not rotate. The object moves in such a way that the rope slowly winds up around the cylinder. The rope will break when the tension exceeds $T_{max}$. Express your answers in terms of $T_{max}$, $m$, $L_0$, $R$, and $v_0$.



What is the length (not yet wound) of the rope?

**(A)** $L_0 - \pi R$

**(B)** $L_0 - 2\pi R$

**(C)** $L_0 - \sqrt{18}\pi R$

**(D)** $\frac{mv_0^2}{T_{max}}$

**(E)** none of the above

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**37** A massless elastic cord (that obeys Hooke's Law) will break if the tension in the cord exceeds $T_{max}$. One end of the cord is attached to a fixed point, the other is attached to an object of mass $3m$. If a second, smaller object of mass m moving at an initial speed $v_0$ strikes the larger mass and the two stick together, the cord will stretch and break, but the final kinetic energy of the two masses will be zero. If instead the two collide with a perfectly elastic one-dimensional collision, the cord will still break, and the larger mass will move off with a final speed of $v_f$. All motion occurs on a horizontal, frictionless surface.

Find $v_f/v_0$.

**(A)** $1/\sqrt{12}$ **(B)** $1/\sqrt{2}$ **(C)** $1/\sqrt{6}$ **(D)** $1/\sqrt{3}$ **(E)** none of the above

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**38** A massless elastic cord (that obeys Hooke's Law) will break if the tension in the cord exceeds $T_{max}$. One end of the cord is attached to a fixed point, the other is attached to an object of mass $3m$. If a second, smaller object of mass m moving at an initial speed $v_0$ strikes the larger mass and the two stick together, the cord will stretch and break, but the final kinetic energy of the two masses will be zero. If instead the two collide with a perfectly elastic one-dimensional collision, the cord will still break, and the larger mass will move off with a final speed of $v_f$. All motion occurs on a horizontal, frictionless surface.

Find the ratio of the total kinetic energy of the system of two masses after the perfectly elastic collision and the cord has broken to the initial kinetic energy of the smaller mass prior to the collision.

**(A)** $1/4$ **(B)** $1/3$ **(C)** $1/2$ **(D)** $3/4$ **(E)** none of the above