Spain Mathematical Olympiad 2017

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by whiwho

1 Find the amount of different values given by the following expression:

$\frac{n^2-2}{n^2-n+2}$

where $n \in \{1,2,3,..,100\}$

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2 A midpoint plotter is an instrument which draws the exact mid point of two point previously drawn. Starting off two points 1 unit of distance apart and using only the midpoint plotter, you have to get two point which are strictly at a distance between $\frac{1}{2017}$ and $\frac{1}{2016}$ units, drawing the minimum amount of points. Which is the minimum number of times you will need to use the midpoint plotter and what strategy should you follow to achieve it?

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3 Let p be an odd prime and $S_q = \frac{1}{2*3*4} + \frac{1}{5*6*7} + ... + \frac{1}{q(q+1)(q+2)}$, where $q = \frac{3p-5}{2}$. We write $\frac{1}{2} - 2S_q$ in the form $\frac{m}{n}$, where m and n are integers. Prove that $m \equiv n (mod p)$

4

You are given a row made by 2018 squares, numbered consecutively from 0 to 2017. Initially, there is a coin in the square 0. Two players A and B play alternatively, starting with A, on the following way: In his turn, each player can either make his coin advance 53 squares or make the coin go back 2 squares. On each move the coin can never go to a number less than 0 or greater than 2017. The player who puts the coin on the square 2017 wins. Who is the one with the wining strategy and how should he play to win?

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5 Let a, b, c be positive real numbers so that $a + b + c = \frac{1}{\sqrt{3}}$. Find the maximum value of

$$27abc + a\sqrt{a^2+2bc} + b\sqrt{b^2+2ca} + c\sqrt{c^2+2ab}.$$

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6 In the triangle ABC, the respective mid points of the sides BC, AB and AC are D, E and F. Let M be the point where the internal bisector of the angle $\angle ADB$ intersects the side AB, and N the point where the internal bisector of the angle $\angle ADC$ intersects the side AC. Also, let O be the intersection point of AD and MN, P the intersection point of AB and FO, and R the intersection point of AC and EO. Prove that $PR = AD$.
