USAPhO 2008

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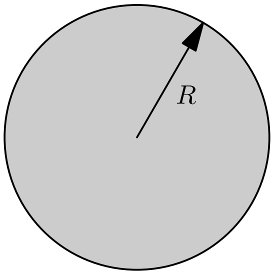
– Quarterfinal

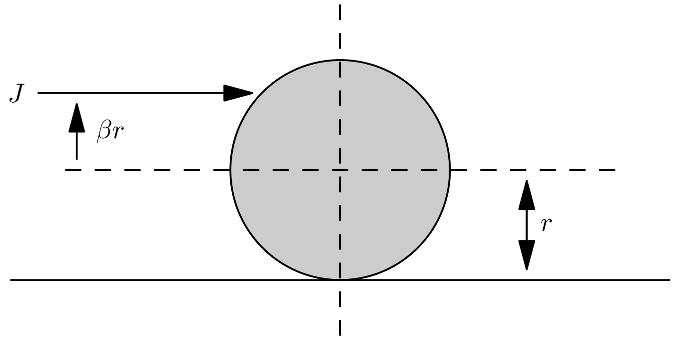
1 A charged particle with charge q and mass m is given an initial kinetic energy K_0 at the middle of a uniformly charged spherical region of total charge Q and radius R. q and Q have opposite signs. The spherically charged region is not free to move. Throughout this problem consider electrostatic forces only.



(a) Find the value of K_0 such that the particle will just reach the boundary of the spherically charged region.

(b) How much time does it take for the particle to reach the boundary of the region if it starts with the kinetic energy K_0 found in part (a)?

2 A uniform pool ball of radius r and mass m begins at rest on a pool table. The ball is given a horizontal impulse J of fixed magnitude at a distance βr above its center, where $-1 \leq \beta \leq 1$. The coefficient of kinetic friction between the ball and the pool table is μ. You may assume the ball and the table are perfectly rigid. Ignore effects due to deformation. (The moment of inertia about the center of mass of a solid sphere of mass m and radius r is $I_{cm} = \frac{2}{5}mr^2$.)



(a) Find an expression for the final speed of the ball as a function of J, m, and β.

(b) For what value of β does the ball immediately begin to roll without slipping, regardless of the value of μ?

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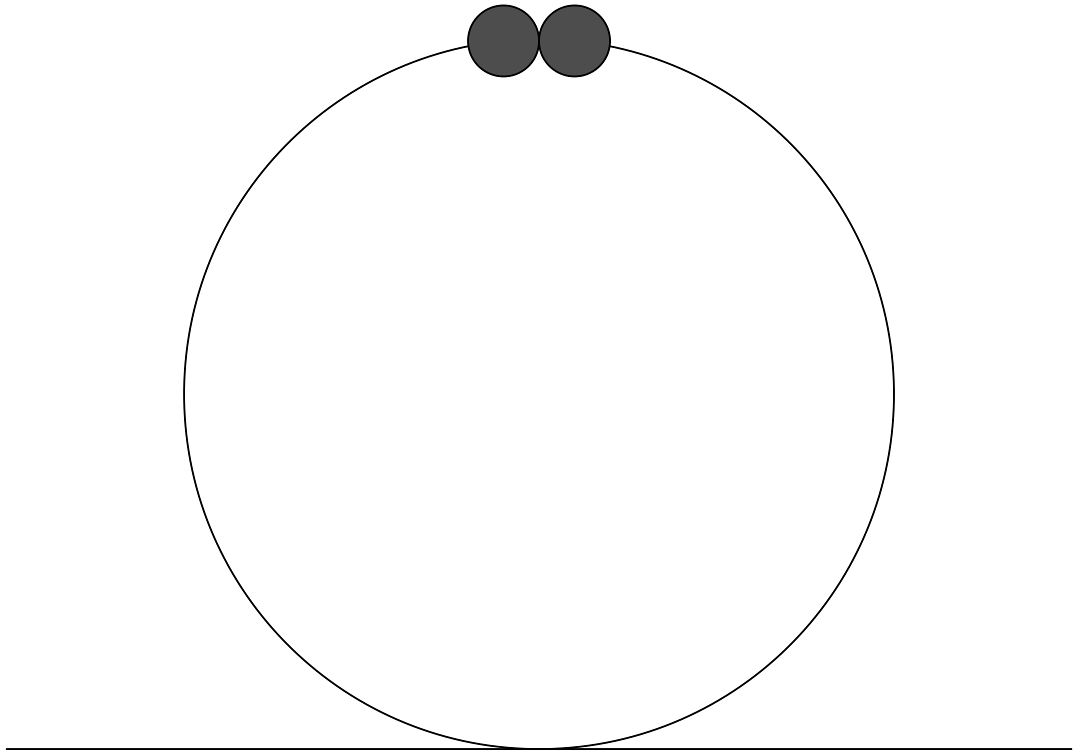
4 Two beads, each of mass m, are free to slide on a rigid, vertical hoop of mass m_h. The beads are threaded on the hoop so that they cannot fall off of the hoop. They are released with negligible velocity at the top of the hoop and slide down to the bottom in opposite directions. The hoop remains vertical at all times. What is the maximum value of the ratio m/m_h such that the hoop always remains in contact with the ground? Neglect friction.



– Semifinal