**Mathematical Olympiad 2011**

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by thugzmath10

**1** Find all nonempty finite sets $X$ of real numbers such that for all $x \in X$, $x + |x| \in X$.

**2** In triangle $ABC$, let $X$ and $Y$ be the midpoints of $AB$ and $AC$, respectively. On segment $BC$, there is a point $D$, different from its midpoint, such that $\angle XDY = \angle BAC$. Prove that $AD \perp BC$.

**3** The $2011$th prime number is $17483$ and the next prime is $17489$.
Does there exist a sequence of $2011^{2011}$ consecutive positive integers that contain exactly $2011$ prime numbers?

**4** Find all (if there is one) functions $f : \mathbb{R} \to \mathbb{R}$ such that for all $x \in \mathbb{R}$,

$$f(f(x)) + xf(x) = 1.$$

**5** The chromatic number $\chi$ of an (infinite) plane is the smallest number of colors with which we can color the points on the plane in such a way that no two points of the same color are one unit apart.
Prove that $4 \leq \chi \leq 7$.