Mathematical Olympiad 2013

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1 1. Determine, with proof, the least positive integer n for which there exist n distinct positive integers, $\left(1-\frac{1}{x_1}\right)\left(1-\frac{1}{x_2}\right)......\left(1-\frac{1}{x_n}\right)=\frac{15}{2013}$

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2 2. Let P be a point in the interior of triangle ABC . Extend AP, BP, and CP to meet BC, AC, and AB at D, E, and F, respectively. If triangle APF, triangle BPD and triangle CPE have equal areas, prove that P is the centroid of triangle ABC .

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3 3. Let n be a positive integer. The numbers 1, 2, 3,....., 2n are randomly assigned to 2n distinct points on a circle. To each chord joining two of these points, a value is assigned equal to the absolute value of the difference between the assigned numbers at its endpoints.
Show that one can choose n pairwise non-intersecting chords such that the sum of the values assigned to them is n^2 .

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4 4. Let a, p and q be positive integers with $p \leq q$. Prove that if one of the numbers a^p and a^q is divisible by p , then the other number must also be divisible by p .

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5 Let r and s be positive real numbers such that $(r+s-rs)(r+s+rs)=rs$. Find the minimum value of $r+s-rs$ and $r+s+rs$
