Greece Team Selection Test 2010

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1 Solve in positive reals the system: $x+y+z+w=4$ $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}+\frac{1}{w}=5-\frac{1}{xyzw}$

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2 In a blackboard there are K circles in a row such that one of the numbers $1,...,K$ is assigned to each circle from the left to the right.
Change of situation of a circle is to write in it or erase the number which is assigned to it.At the beginning no number is written in its own circle.
For every positive divisor d of K ,$1 \leq d \leq K$ we change the situation of the circles in which their assigned numbers are divisible by d,performing for each divisor d K changes of situation.
Determine the value of K for which the following holds;when this procedure is applied once for all positive divisors of K ,then all numbers $1,2,3,...,K$ are written in the circles they were assigned in.

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3 Let ABC be a triangle,O its circumcenter and R the radius of its circumcircle.Denote by O_1 the symmetric of O with respect to BC,O_2 the symmetric of O with respect to AC and by O_3 the symmetric of O with respect to AB.
(a)Prove that the circles $C_1(O_1,R)$, $C_2(O_2,R)$, $C_3(O_3,R)$ have a common point.
(b)Denote by T this point.Let l be an arbitary line passing through T which intersects C_1 at L, C_2 at M and C_3 at K.From K,L,M drop perpendiculars to AB,BC,AC respectively.Prove that these perpendiculars pass through a point.

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4 Find all functions $f:\mathbb{R}^*\rightarrow\mathbb{R}^*$ satisfying $f(\frac{f(x)}{f(y)})=\frac{1}{y}f(f(x))$ for all $x,y\in\mathbb{R}^*$
and are strictly monotone in $(0,+\infty)$
