Manhattan Mathematical Olympiad 2001

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–	Grades 5-6
1	Piglet added together three consecutive whole numbers, then the next three numbers, and multiplied one sum by the other. Could the product be equal to $111,111,111$?
2	The dates of three Sundays of a month were even numbers. What day of the week was the 20th of the month?
3	Is it possible to divide 5 apples of the same size equally between six children so that no apple will be cut into more than 3 pieces? (You are allowed to cut an apple into any number of equal pieces).
4	You have a four-liter jug and a six-liter pot (both of cylindrical shape), and a big barrel of water. Can you measure exactly one liter of water?
–	Grades 7-8
1	The product of a million whole numbers is equal to million. What can be the greatest possible value of the sum of these numbers?
2	There are 2001 marked points in the plane. It is known that the area of any triangle with vertices at the given points is less than or equal than $1\ cm^2$. Prove that there exists a triangle with area no more than $4\ cm^2$, which contains all 2001 points.
3	Integer numbers x, y, z satisfy the equation $$x^3 + y^3 = z^3.$$ Prove that at least one of them is divisible by 3.
4	You have a pencil, paper and an angle of 19 degrees made out of two equal very thin sticks. Can you construct an angle of 1 degree using only these tools?
–	Grades 9-12
1	Find all integer solutions to the equation $$x^2 + y^2 + z^2 = 2xyz$$

2 Prove that circles which have sides of a convex quadrilateral as diameters cover its interior. (Convex polygon is the one which contains with any two points the whole segment, joining them).

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3 Let x_1 and x_2 be roots of the equation $x^2 - 6x + 1 = 0$. Prove that for any integer $n \geq 1$ the number $x_1^n + x_2^n$ is integer and is not divisible by 5.

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4 How many digits has the number 2^{100}?

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5 Factorize the expression $a^3 + b^3 + c^3 - 3abc$.

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6 There are n coins of the radius r on a table which is a circle of the radius R. It is known that:

a) any two coins either touch each other or have no common points;
b) there is no place for one more coin on the table.

Prove that

$$\frac{1}{2}\left(\frac{R}{r} - 1\right) < \sqrt{n} < \frac{R}{r}.$$
