Nordic 2005

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by arccosinus

1 Find all positive integers k such that the product of the digits of k, in decimal notation, equals

$$\frac{25}{8}k - 211$$

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2 Let a, b, c be positive real numbers. Prove that

$$\frac{2a^2}{b+c} + \frac{2b^2}{c+a} + \frac{2c^2}{a+b} \geq a+b+c$$

(this is, of course, a joke!)

EDITED with exponent 2 over c

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3 There are 2005 young people sitting around a large circular table. Of these, at most 668 are boys. We say that a girl G has a strong position, if, counting from G in either direction, the number of girls is always strictly larger than the number of boys (G is herself included in the count). Prove that there is always a girl in a strong position.

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4 The circle ζ_1 is inside the circle ζ_2, and the circles touch each other at A. A line through A intersects ζ_1 also at B, and ζ_2 also at C. The tangent to ζ_1 at B intersects ζ_2 at D and E. The tangents of ζ_1 passing thorugh C touch ζ_2 at F and G. Prove that D, E, F and G are concyclic.
