Nordic 2006

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by ACCCGS8

1 Points B, C vary on two fixed rays emanating from point A such that $AB + AC$ is constant. Show that there is a point D, other than A, such that the circumcircle of triangle ABC passes through D for all possible choices of B, C.

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2 Real numbers x, y, z are not all equal and satisfy $x + \frac{1}{y} = y + \frac{1}{z} = z + \frac{1}{x} = k$. Find all possible values of k.

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3 A sequence (a_n) of positive integers is defined by $a_0 = m$ and $a_{n+1} = a_n^5 + 487$ for all $n \geq 0$. Find all positive integers m such that the sequence contains the maximum possible number of perfect squares.

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4 Each square of a 100×100 board is painted with one of 100 different colours, so that each colour is used exactly 100 times. Show that there exists a row or column of the chessboard in which at least 10 colours are used.
