Nordic 2007

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by ACCCGS8

1   Find a solution to the equation $x^2 - 2x - 2007y^2 = 0$ in positive integers.

2   Three given rectangles cover the sides of a triangle completely and each rectangle has a side parallel to a given line. Show that the rectangles also cover the interior of the triangle.

3   The number 10^{2007} is written on the blackboard. Anne and Berit play a two player game in which the player in turn performs one of the following operations:
1) replace a number x on the blackboard with two integers $a, b > 1$ such that $ab = x$.
2) strike off one or both of two equal numbers on the blackboard.

The person who cannot perform any operation loses.
Who has the winning strategy if Anne starts?

4   A line through A intersects a circle at points B, C with B between A, C. The two tangents from A intersect the circle at S, T. ST and AC intersect at P. Show that $\frac{AP}{PC} = 2\frac{AB}{BC}$.