Nordic 2008

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by ACCCGS8

1 Find all reals A, B, C such that there exists a real function f satisfying $f(x+f(y)) = Ax + By + C$ for all reals x, y.

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2 Assume that $n \geq 3$ people with different names sit around a round table. We call any unordered pair of them, say M, N, dominating if
1) they do not sit in adjacent seats
2) on one or both arcs connecting M, N along the table, all people have names coming alphabetically after M, N.

Determine the minimal number of dominating pairs.

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3 Let ABC be a triangle and D, E be points on BC, CA such that AD, BE are angle bisectors of $\triangle ABC$. Let F, G be points on the circumcircle of $\triangle ABC$ such that $AF || DE$ and $FG || BC$. Prove that $\frac{AG}{BG} = \frac{AB+AC}{AB+BC}$.

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4 The difference between the cubes of two consecutive positive integers is equal to n^2 for a positive integer n. Show that n is the sum of two squares.
