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1 When $a_0, a_1, \ldots, a_{1000}$ denote digits, can the sum of the 1001-digit numbers $a_0a_1\cdots a_{1000}$ and $a_{1000}a_{999}\cdots a_0$ have odd digits only?

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2 In a triangle ABC assume $AB = AC$, and let D and E be points on the extension of segment BA beyond A and on the segment BC, respectively, such that the lines CD and AE are parallel. Prove $CD \geq \frac{4h}{BC}CE$, where h is the height from A in triangle ABC. When does equality hold?

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3 Find all functions f such that

$$f(f(x)+y) = f(x^2-y) + 4yf(x)$$

for all real numbers x and y.

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4 Show that for any integer $n \geq 2$ the sum of the fractions $\frac{1}{ab}$, where a and b are relatively prime positive integers such that $a < b \leq n$ and $a+b > n$, equals $\frac{1}{2}$.
(Integers a and b are called relatively prime if the greatest common divisor of a and b is 1.)
