Greece National Olympiad 2017

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1 An acute triangle ABC with $AB < AC < BC$ is inscribed in a circle $c(O, R)$. The circle $c_1(A, AC)$ intersects the circle c at point D and intersects CB at E. If the line AE intersects c at F and G lies in BC such that $EB = BG$, prove that F, E, D, G are concyclic.

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2 Let A be a point in the plane and 3 lines which pass through this point divide the plane in 6 regions.
In each region there are 5 points. We know that no three of the 30 points existing in these regions are collinear. Prove that there exist at least 1000 triangles whose vertices are points of those regions such that A lies either in the interior or on the side of the triangle.

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3 Find all integer triples (a, b, c) with $a > 0 > b > c$ whose sum equal 0 such that the number

$$N = 2017 - a^3b - b^3c - c^3a$$

is a perfect square of an integer.

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4 Let u be the positive root of the equation $x^2 + x - 4 = 0$. The polynomial

$$P(x) = a_nx^n + a_{n-1}x^{n-1} + ... + a_0$$

where n is positive integer has non-negative integer coefficients and $P(u) = 2017$.
1) Prove that $a_0 + a_1 + ... + a_n \equiv 1 \mod 2$.
2) Find the minimum possible value of $a_0 + a_1 + ... + a_n$.
