Greece Team Selection Test 2017

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by Borbas

1 Let ABC be an acute-angled triangle inscribed in circle $c(O,R)$ with $AB<AC<BC$, and c_1 be the inscribed circle of ABC which intersects AB,AC,BC at F,E,D respectivelly. Let A',B',C' be points which lie on c such that the quadrilaterals $AEFA',BDFB',CDEC'$ are inscribable.
(1) Prove that $DEA'B'$ is inscribable.
(2) Prove that DA',EB',FC' are concurrent.

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2 Prove that the number $A=\frac{(4n)!}{(2n)!n!}$ is an integer and divisible by 2^{n+1}, where n is a positive integer.

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3 Find all fuctions $f,g:\mathbb{R}\rightarrow\mathbb{R}$ such that: $f(x-3f(y))=xf(y)-yf(x)+g(x)\forall x,y\in\mathbb{R}$ and $g(1)=-8$

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4 Some positive integers are initially written on a board, where each 2 of them are different.
Each time we can do the following moves:
(1) If there are 2 numbers (written in the board) in the form $n,n+1$ we can erase them and write down $n-2$
(2) If there are 2 numbers (written in the board) in the form $n,n+4$ we can erase them and write down $n-1$
After some moves, there might appear negative numbers. Find the maximum value of the integer c such that:
Independetly of the starting numbers, each number which appears in any move is greater or equal to c

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