

**German National Olympiad 2016, Final Round**[www.artofproblemsolving.com/community/c447162](http://www.artofproblemsolving.com/community/c447162)

by Tintarn, Kezer

## – Day 1

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- 1 Find all real pairs  $(a, b)$  that solve the system of equation

$$\begin{aligned}a^2 + b^2 &= 25, \\ 3(a + b) - ab &= 15.\end{aligned}$$

(German MO 2016 - Problem 1)

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- 2 A very well known family of mathematicians has three children called *Antonia*, *Bernhard* and *Christian*. Each evening one of the children has to do the dishes. One day, their dad decided to construct of plan that says which child has to do the dishes at which day for the following 55 days.

Let  $x$  be the number of possible such plans in which Antonia has to do the dishes on three consecutive days at least once. Furthermore, let  $y$  be the number of such plans in which there are three consecutive days in which Antonia does the dishes on the first, Bernhard on the second and Christian on the third day.

Determine, whether  $x$  and  $y$  are different and if so, then decide which of those is larger.

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- 3 Let  $I_a$  be the  $A$ -excenter of a scalene triangle  $ABC$ . And let  $M$  be the point symmetric to  $I_a$  about line  $BC$ .  
Prove that line  $AM$  is parallel to the line through the circumcenter and the orthocenter of triangle  $I_aCB$ .

## – Day 2

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- 4 Find all positive integers  $m, n$  with  $m \leq 2n$  that solve the equation

$$m \cdot \binom{2n}{n} = \binom{m^2}{2}.$$

(German MO 2016 - Problem 4)

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- 5 Let  $A, B, C, D$  be points on a circle with radius  $r$  in this order such that  $|AB| = |BC| = |CD| = s$  and  $|AD| = s + r$ . Find all possible values of the interior angles of the quadrilateral  $ABCD$ .
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6 Let

$$f(x_1, x_2, x_3, x_4, x_5, x_6, x_7) = x_1x_2x_4 + x_2x_3x_5 + x_3x_4x_6 + x_4x_5x_7 + x_5x_6x_1 + x_6x_7x_2 + x_7x_1x_3$$

be defined for non-negative real numbers  $x_1, x_2, \dots, x_7$  with sum 1.

Prove that  $f(x_1, x_2, \dots, x_7)$  has a maximum value and find that value.

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