

**Balkan MO 2017**

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by sqing, Ferid. —, Snakes

- 1 Find all ordered pairs of positive integers  $(x, y)$  such that:

$$x^3 + y^3 = x^2 + 42xy + y^2.$$

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- 2 Consider an acute-angled triangle  $ABC$  with  $AB < AC$  and let  $\omega$  be its circumscribed circle. Let  $t_B$  and  $t_C$  be the tangents to the circle  $\omega$  at points  $B$  and  $C$ , respectively, and let  $L$  be their intersection. The straight line passing through the point  $B$  and parallel to  $AC$  intersects  $t_C$  in point  $D$ . The straight line passing through the point  $C$  and parallel to  $AB$  intersects  $t_B$  in point  $E$ . The circumcircle of the triangle  $BDC$  intersects  $AC$  in  $T$ , where  $T$  is located between  $A$  and  $C$ . The circumcircle of the triangle  $BEC$  intersects the line  $AB$  (or its extension) in  $S$ , where  $B$  is located between  $S$  and  $A$ . Prove that  $ST$ ,  $AL$ , and  $BC$  are concurrent.

Vangelis Psychas and Silouanos Brazitikos

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- 3 Let  $\mathbb{N}$  denote the set of positive integers. Find all functions  $f : \mathbb{N} \rightarrow \mathbb{N}$  such that

$$n + f(m) \mid f(n) + nf(m)$$

for all  $m, n \in \mathbb{N}$

*Proposed by Dorlir Ahmeti, Albania*

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- 4 On a circular table sit  $n > 2$  students. First, each student has just one candy. At each step, each student chooses one of the following actions:

(A) Gives a candy to the student sitting on his left or to the student sitting on his right.

(B) Separates all its candies in two, possibly empty, sets and gives one set to the student sitting on his left and the other to the student sitting on his right.

At each step, students perform the actions they have chosen at the same time.

A distribution of candy is called legitimate if it can occur after a finite number of steps.

Find the number of legitimate distributions.

(Two distributions are different if there is a student who has a different number of candy in each of these distributions.)

Forgive my poor English