Junior Balkan Team Selection Tests - Moldova 2017
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by socrates

Problem 1 Find all natural numbers x, y such that

$$x^5 = y^5 + 10y^2 + 20y + 1.$$

Problem 2 Let a, b, c be the sidelengths of a triangle. Prove that

$$2 < \sqrt{\frac{a}{b+c}} + \sqrt{\frac{b}{c+a}} + \sqrt{\frac{c}{a+b}} < \sqrt{6}.$$

Problem 3 Let ABC be a triangle inscribed in a semicircle with center O and diameter BC. Two tangent lines to the semicircle at A and B intersect at D. Prove that DC goes through the midpoint of the altitude AH of triangle ABC.

Problem 4 Find the maximum positive integer k such that there exist k positive integers which do not exceed 2017 and have the property that every number among them cannot be a power of any of the remaining $k-1$ numbers.

Problem 5 Consider the following increasing sequence $1, 3, 5, 7, 9,$ of all positive integers consisting only of odd digits. Find the 2017 -th term of the above sequence.

Problem 6 Let a, b and c be real numbers such that $|a+b| + |b+c| + |c+a| = 8.$ Find the maximum and minimum value of the expression $P = a^2 + b^2 + c^2$.

Problem 7 Given is an acute triangle ABC and the median AM. Draw $BH \perp AC$. The line which goes through A and is perpendicular to AM intersects BH at E. On the opposite ray of the ray AE choose F such that $AE = AF$. Prove that $CF \perp AB$.

Problem 8 The bottom line of a 2×13 rectangle is filled with 13 tokens marked with the numbers $1, 2, ..., 13$ and located in that order. An operation is a move of a token from its cell into a free adjacent cell (two cells are called adjacent if they have a common side). What is the minimum number of operations needed to rearrange the chips in reverse order in the bottom line of the rectangle?
