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by gauss1181, Binomial-theorem, rrusczyk

1 The product of two of the four roots of the quartic equation $x^4 - 18x^3 + kx^2 + 200x - 1984 = 0$ is -32. Determine the value of k.

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2 The geometric mean of any set of m non-negative numbers is the m-th root of their product.

(i) For which positive integers n is there a finite set S_n of n distinct positive integers such that the geometric mean of any subset of S_n is an integer? (ii) Is there an infinite set S of distinct positive integers such that the geometric mean of any finite subset of S is an integer?

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3 $P, A, B, C,$ and D are five distinct points in space such that $\angle APB = \angle BPC = \angle CPD = \angle DPA = \theta$, where θ is a given acute angle. Determine the greatest and least values of $\angle APC + \angle BPD$.

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4 A difficult mathematical competition consisted of a Part I and a Part II with a combined total of 28 problems. Each contestant solved 7 problems altogether. For each pair of problems, there were exactly two contestants who solved both of them. Prove that there was a contestant who, in Part I, solved either no problems or at least four problems.

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5 $P(x)$ is a polynomial of degree $3n$ such that

$$\begin{aligned} P(0) = P(3) = \cdots &= P(3n) = 2, \\ P(1) = P(4) = \cdots &= P(3n-2) = 1, \\ P(2) = P(5) = \cdots &= P(3n-1) = 0, \quad \text{and} \\ & P(3n+1) = 730. \end{aligned}$$

Determine n.

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