Olympic Revenge 2017
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1 Prove that does not exist positive integers a, b and k such that $4abk-a-b$ is a perfect square.

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2 Let $\triangle ABC$ a triangle with circumcircle Γ. Suppose there exist points R and S on sides AB and AC, respectively, such that $BR=RS=SC$. A tangent line through A to Γ meet the line RS at P. Let I the incenter of triangle $\triangle ARS$. Prove that $PA=PI$

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3 Let n a positive integer. We call a pair (π, C) composed by a permutation π: $1,2,...n \to 1,2,...,n$ and a binary function $C: 1,2,...,n \to 0,1$ "revengeful" if it satisfies the two following conditions:

$1)$For every $i \in 1,2,...,n$, there exist $j \in S_i = i, \pi(i), \pi(\pi(i)), ...$ such that $C(j)=1$.

$2)$ If $C(k)=1$, then k is one of the $v_2(|S_k|)+1$ highest elements of S_k, where $v_2(t)$ is the highest nonnegative integer such that $2^{v_2(t)}$ divides t, for every positive integer t.

Let V the number of revengeful pairs and P the number of partitions of n with all parts powers of 2.
Determine $\frac{V}{P}$.

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4 Let $f: \mathbb{R}_+^* \to \mathbb{R}_+^*$ such that $f'''(x) > 0$ for all $x \in \mathbb{R}_+^*$. Prove that:

$f(a^2+b^2+c^2)+2f(ab+bc+ac) \geq f(a^2+2bc)+f(b^2+2ca)+f(c^2+2ab)$, for all $a,b,c \in \mathbb{R}_+^*$.
