

**USAMO 2001**

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**Day 1** May 1st

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**1** Each of eight boxes contains six balls. Each ball has been colored with one of  $n$  colors, such that no two balls in the same box are the same color, and no two colors occur together in more than one box. Determine, with justification, the smallest integer  $n$  for which this is possible.

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**2** Let  $ABC$  be a triangle and let  $\omega$  be its incircle. Denote by  $D_1$  and  $E_1$  the points where  $\omega$  is tangent to sides  $BC$  and  $AC$ , respectively. Denote by  $D_2$  and  $E_2$  the points on sides  $BC$  and  $AC$ , respectively, such that  $CD_2 = BD_1$  and  $CE_2 = AE_1$ , and denote by  $P$  the point of intersection of segments  $AD_2$  and  $BE_2$ . Circle  $\omega$  intersects segment  $AD_2$  at two points, the closer of which to the vertex  $A$  is denoted by  $Q$ . Prove that  $AQ = D_2P$ .

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**3** Let  $a, b, c \geq 0$  and satisfy

$$a^2 + b^2 + c^2 + abc = 4.$$

Show that

$$0 \leq ab + bc + ca - abc \leq 2.$$

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**Day 2** May 2nd

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**4** Let  $P$  be a point in the plane of triangle  $ABC$  such that the segments  $PA$ ,  $PB$ , and  $PC$  are the sides of an obtuse triangle. Assume that in this triangle the obtuse angle opposes the side congruent to  $PA$ . Prove that  $\angle BAC$  is acute.

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**5** Let  $S$  be a set of integers (not necessarily positive) such that

(a) there exist  $a, b \in S$  with  $\gcd(a, b) = \gcd(a - 2, b - 2) = 1$ ;

(b) if  $x$  and  $y$  are elements of  $S$  (possibly equal), then  $x^2 - y$  also belongs to  $S$ .

Prove that  $S$  is the set of all integers.

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**6** Each point in the plane is assigned a real number such that, for any triangle, the number at the center of its inscribed circle is equal to the arithmetic mean of the three numbers at its vertices. Prove that all points in the plane are assigned the same number.

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