Pan African 2000

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Day 1

1 Solve for $x \in R$:

$$\sin^3 x(1+\cot x)+\cos^3 x(1+\tan x)=\cos 2x$$

2 Define the polynomials $P_0, P_1, P_2 \cdots$ by:

$$P_0(x)=x^3+213x^2-67x-2000$$

$$P_n(x)=P_{n-1}(x-n), n \in N$$

Find the coefficient of x in $P_{21}(x)$.

3 Let p and q be coprime positive integers such that:

$$\frac{p}{q}=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}\cdots-\frac{1}{1334}+\frac{1}{1335}$$

Prove p is divisible by 2003.

Day 2

1 Let a, b and c be real numbers such that $a^2+b^2=c^2$, solve the system:

$$z^2=x^2+y^2$$

$$(z+c)^2=(x+a)^2+(y+b)^2$$

in real numbers x, y and z.

2 Let γ be circle and let P be a point outside γ. Let PA and PB be the tangents from P to γ (where $A, B \in \gamma$). A line passing through P intersects γ at points Q and R. Let S be a point on γ such that $BS \parallel QR$. Prove that SA bisects QR.

3 A company has five directors. The regulations of the company require that any majority (three or more) of the directors should be able to open its strongroom, but any minority (two or less) should not be able to do so. The strongroom is equipped with ten locks, so that it can only be opened when keys to all ten locks are available. Find all positive integers n such that it is possible to give each of the directors a set of keys to n different locks, according to the requirements and regulations of the company.

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