

**National Math Olympiad (Second Round) 2017**

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by soroush.MG, andria, bgn

- 1 a) Prove that there doesn't exist sequence  $a_1, a_2, a_3, \dots \in \mathbb{N}$  such that:  $\forall i < j : \gcd(a_i + j, a_j + i) = 1$
- b) Let  $p$  be an odd prime number. Prove that there exist sequence  $a_1, a_2, a_3, \dots \in \mathbb{N}$  such that:  $\forall i < j : p \nmid \gcd(a_i + j, a_j + i)$
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- 2 Let  $ABCD$  be an isosceles trapezoid such that  $AB \parallel CD$ . Suppose that there exists a point  $P$  in  $ABCD$  such that  $\angle APB > \angle ADC$  and  $\angle DPC > \angle ABC$ . Prove that

$$AB + CD > DA + BC.$$

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- 3 Let  $n$  be a natural number divisible by 3. We have a  $n \times n$  table and each square is colored either black or white. Suppose that for all  $m \times m$  sub-tables from the table ( $m > 1$ ), the number of black squares is not more than white squares. Find the maximum number of black squares.
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- 4 Let  $x, y$  be two positive real numbers such that  $x^4 - y^4 = x - y$ . Prove that

$$\frac{x - y}{x^6 - y^6} \leq \frac{4}{3}(x + y).$$

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- 5 There are five smart kids sitting around a round table. Their teacher says: "I gave a few apples to some of you, and none of you have the same amount of apple. Also each of you will know the amount of apple that the person to your left and the person to your right has." The teacher tells the total amount of apples, then asks the kids to guess the difference of the amount of apple that the two kids in front of them have.
- a) If the total amount of apples is less than 16, prove that at least one of the kids will guess the difference correctly. b) Prove that the teacher can give the total of 16 apples such that no one can guess the difference correctly.
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- 6 Let  $ABC$  be a triangle and  $X$  be a point on its circumcircle.  $Q, P$  lie on a line  $BC$  such that  $XQ \perp AC, XP \perp AB$ . Let  $Y$  be the circumcenter of  $\triangle XQP$ . Prove that  $ABC$  is equilateral triangle if and only if  $Y$  moves on a circle when  $X$  varies on the circumcircle of  $ABC$ .
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