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by BartSimpsons

1 We call a 5-tuple of integers *arrangeable* if its elements can be labeled a, b, c, d, e in some order so that $a-b+c-d+e=29$. Determine all 2017-tuples of integers $n_1, n_2, ..., n_{2017}$ such that if we place them in a circle in clockwise order, then any 5-tuple of numbers in consecutive positions on the circle is arrangeable.

Warut Suksompong, Thailand

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2 Let ABC be a triangle with $AB < AC$. Let D be the intersection point of the internal bisector of angle BAC and the circumcircle of ABC. Let Z be the intersection point of the perpendicular bisector of AC with the external bisector of angle $\angle BAC$. Prove that the midpoint of the segment AB lies on the circumcircle of triangle ADZ.

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3 Let $A(n)$ denote the number of sequences $a_1 \geq a_2 \geq \cdots \geq a_k$ of positive integers for which $a_1 + \cdots + a_k = n$ and each $a_i + 1$ is a power of two $(i = 1, 2, \cdots, k)$. Let $B(n)$ denote the number of sequences $b_1 \geq b_2 \geq \cdots \geq b_m$ of positive integers for which $b_1 + \cdots + b_m = n$ and each inequality $b_j \geq 2b_{j+1}$ holds $(j = 1, 2, \cdots, m-1)$. Prove that $A(n) = B(n)$ for every positive integer n.

Senior Problems Committee of the Australian Mathematical Olympiad Committee

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4 Call a rational number r *powerful* if r can be expressed in the form $\dfrac{p^k}{q}$ for some relatively prime positive integers p, q and some integer $k > 1$. Let a, b, c be positive rational numbers such that $abc = 1$. Suppose there exist positive integers x, y, z such that $a^x + b^y + c^z$ is an integer. Prove that a, b, c are all *powerful*.

Jeck Lim, Singapore

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5 Let n be a positive integer. A pair of n-tuples $(a_1, \cdots, a_n)$ and $(b_1, \cdots, b_n)$ with integer entries is called an *exquisite pair* if

$$|a_1 b_1 + \cdots + a_n b_n| \leq 1.$$

Determine the maximum number of distinct n-tuples with integer entries such that any two of them form an exquisite pair.

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