

Pan African 2004

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by shobber

Day 1

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- 1 Do there exist positive integers m and n such that:

$$3n^2 + 3n + 7 = m^3$$

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- 2 Is:

$$4\sqrt{4 - 2\sqrt{3}} + \sqrt{97 - 56\sqrt{3}}$$

an integer?

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- 3 One writes 268 numbers around a circle, such that the sum of 20 consecutive numbers is always equal to 75. The number 3, 4 and 9 are written in positions 17, 83 and 144 respectively. Find the number in position 210.

Day 2

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- 1 Three real numbers satisfy the following statements:
(1) the square of their sum equals to the sum their squares.
(2) the product of the first two numbers is equal to the square of the third number.

Find these numbers.

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- 2 Each of the digits 1, 3, 7 and 9 occurs at least once in the decimal representation of some positive integers. Prove that one can permute the digits of this integer such that the resulting integer is divisible by 7.

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- 3 Let $ABCD$ be a cyclic quadrilateral such that AB is a diameter of it's circumcircle. Suppose that AB and CD intersect at I , AD and BC at J , AC and BD at K , and let N be a point on AB . Show that IK is perpendicular to JN if and only if N is the midpoint of AB .
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