

Pan African 2005

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Day 1 August 1st

- 1 For any positive real numbers a, b and c , prove:

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \geq \frac{2}{a+b} + \frac{2}{b+c} + \frac{2}{c+a} \geq \frac{9}{a+b+c}$$

- 2 Let S be a set of integers with the property that any integer root of any non-zero polynomial with coefficients in S also belongs to S . If 0 and 1000 are elements of S , prove that -2 is also an element of S .
- 3 Let ABC be a triangle and let P be a point on one of the sides of ABC . Construct a line passing through P that divides triangle ABC into two parts of equal area.

Day 2 August 2nd

- 1 Let $[x]$ be the greatest integer less than or equal to x , and let $\{x\} = x - [x]$. Solve the equation: $[x] \cdot \{x\} = 2005x$
- 2 Noah has to fit 8 species of animals into 4 cages of the Arc. He plans to put two species of animal in each cage. It turns out that, for each species of animal, there are at most 3 other species with which it cannot share a cage. Prove that there is a way to assign the animals to the cages so that each species shares a cage with a compatible species.
- 3 Let $f : \mathbb{Z} \rightarrow \mathbb{Z}$ be a function such that: For all a and b in $\mathbb{Z} - \{0\}$, $f(ab) \geq f(a) + f(b)$. Show that for all $a \in \mathbb{Z} - \{0\}$ we have $f(a^n) = nf(a)$ for all $n \in \mathbb{N}$ if and only if $f(a^2) = 2f(a)$