Pan African 2012

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by ACCCGS8

Day 1

1 AB is a chord (not a diameter) of a circle with centre O. Let T be a point on segment OB. The line through T perpendicular to OB meets AB at C and the circle at D and E. Denote by S the orthogonal projection of T onto AB .
Prove that $AS \cdot BC = TE \cdot TD$.

2 Find all positive integers m and n such that $n^m - m$ divides $m^2 + 2m$.

3 Find all real solutions x to the equation $\lfloor x^2 - 2x \rfloor + 2\lfloor x \rfloor = \lfloor x \rfloor^2$.

Day 2

1 The numbers $\frac{1}{1}, \frac{1}{2}, \cdots, \frac{1}{2012}$ are written on the blackboard. Acha chooses any two numbers from the blackboard, say x and y, erases them and she writes instead the number $x + y + xy$. She continues to do this until only one number is left on the board. What are the possible values of the final number?

2 Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that $f(x^2 - y^2) = (x + y)(f(x) - f(y))$ for all real numbers x and y.

3 (i) Find the angles of $\triangle ABC$ if the length of the altitude through B is equal to the length of the median through C and the length of the altitude through C is equal to the length of the median through B.

(ii) Find all possible values of $\angle ABC$ of $\triangle ABC$ if the length of the altitude through A is equal to the length of the median through C and the length of the altitude through C is equal to the length of the median through B.

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