IberoAmerican 1985

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by carlosbr

Day 1

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1 Find all the triples of integers (a, b, c) such that:

$$\begin{aligned} a+b+c &= 24 \\ a^2+b^2+c^2 &= 210 \\ abc &= 440 \end{aligned}$$

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2 Let P be a point in the interior of the equilateral triangle $\triangle ABC$ such that $PA = 5$, $PB = 7$, $PC = 8$. Find the length of the side of the triangle ABC.

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3 Find all the roots r_1, r_2, r_3 y r_4 of the equation $4x^4 - ax^3 + bx^2 - cx + 5 = 0$, knowing that they are real, positive and that:

$$\frac{r_1}{2} + \frac{r_2}{4} + \frac{r_3}{5} + \frac{r_4}{8} = 1.$$

Day 2

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1 If $x \neq 1$, $y \neq 1$, $x \neq y$ and

$$\frac{yz - x^2}{1 - x} = \frac{xz - y^2}{1 - y}$$

show that both fractions are equal to $x + y + z$.

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2 To each positive integer n it is assigned a non-negative integer $f(n)$ such that the following conditions are satisfied:

(1) $f(rs) = f(r) + f(s)$
(2) $f(n) = 0$, if the first digit (from right to left) of n is 3.
(3) $f(10) = 0$.

Find $f(1985)$. Justify your answer.

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3 Given an acute triangle ABC, let D, E and F be points in the lines BC, AC and AB respectively. If the lines AD, BE and CF pass through O the centre of the circumcircle of the triangle ABC, whose radius is R, show that:

$$\frac{1}{AD} + \frac{1}{BE} + \frac{1}{CF} = \frac{2}{R}$$
