IberoAmerican 1988

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Day 1

1	The measure of the angles of a triangle are in arithmetic progression and the lengths of its altitudes are as well. Show that such a triangle is equilateral.
2	Let a, b, c, d, p and q be positive integers satisfying $ad - bc = 1$ and $\frac{a}{b} > \frac{p}{q} > \frac{c}{d}$. Prove that: (a) $q \geq b + d$ (b) If $q = b + d$, then $p = a + c$.
3	Prove that among all possible triangles whose vertices are $3, 5$ and 7 apart from a given point P, the ones with the largest perimeter have P as incentre.

Day 2

4	$\triangle ABC$ is a triangle with sides a, b, c. Each side of $\triangle ABC$ is divided in n equal segments. Let S be the sum of the squares of the distances from each vertex to each of the points of division on its opposite side. Show that $\frac{S}{a^2+b^2+c^2}$ is a rational number.
5	Consider all the numbers of the form $x + yt + zt^2$, with x, y, z rational numbers and $t = \sqrt[3]{2}$. Prove that if $x + yt + zt^2 \neq 0$, then there exist rational numbers u, v, w such that $$(x + yt + z^2)(u + vt + wt^2) = 1$$
6	Consider all sets of n distinct positive integers, no three of which form an arithmetic progression. Prove that among all such sets there is one which has the largest sum of the reciprocals of its elements.