IberoAmerican 1989

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by WakeUp

Day 1

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1 Determine all triples of real numbers that satisfy the following system of equations:

$$x + y - z = -1x^2 - y^2 + z^2 = 1 - x^3 + y^3 + z^3 = -1$$

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2 Let x, y, z be real numbers such that $0 \le x, y, z \le \frac{\pi}{2}$. Prove the inequality

$$\frac{\pi}{2} + 2\sin x \cos y + 2\sin y \cos z \ge \sin 2x + \sin 2y + \sin 2z.$$

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3 Let a, b and c be the side lengths of a triangle. Prove that:

$$\frac{a-b}{a+b} + \frac{b-c}{b+c} + \frac{c-a}{c+a} < \frac{1}{16}$$

Day 2

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1 The incircle of the triangle ABC is tangent to sides AC and BC at M and N, respectively. The bisectors of the angles at A and B intersect MN at points P and Q, respectively. Let O be the incentre of $\triangle ABC$. Prove that $MP \cdot OA = BC \cdot OQ$.

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2 Let the function f be defined on the set $\mathbb{N}$ such that

(i) $f(1) = 1$ (ii) $f(2n+1) = f(2n) + 1$ (iii) $f(2n) = 3f(n)$

Determine the set of values taken f.

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3 Show that the equation $2x^2 - 3x = 3y^2$ has infinitely many solutions in positive integers.
