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1 Each vertex of a cube is assigned an 1 or a -1, and each face is assigned the product of the numbers assigned to its vertices. Determine the possible values the sum of these 14 numbers can attain.

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2 A square is divided in four parts by two perpendicular lines, in such a way that three of these parts have areas equal to 1. Show that the square has area equal to 4.

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3 Let $f:\ [0,\ 1]\rightarrow\mathbb{R}$ be an increasing function satisfying the following conditions:

a) $f(0)=0$;

b) $f\left(\frac{x}{3}\right)=\frac{f(x)}{2}$;

c) $f(1-x)=1-f(x)$.

Determine $f\left(\frac{18}{1991}\right)$.

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4 Find a positive integer n with five non-zero different digits, which satisfies to be equal to the sum of all the three-digit numbers that can be formed using the digits of n.

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5 Let $P(x,\ y)=2x^2-6xy+5y^2$. Let us say an integer number a is a value of P if there exist integer numbers b, c such that $P(b,\ c)=a$.

a) Find all values of P lying between 1 and 100.

b) Show that if r and s are values of P, then so is rs.

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6 Let M, N and P be three non-collinear points. Construct using straight edge and compass a triangle for which M and N are the midpoints of two of its sides, and P is its orthocenter.
