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Day 1 September 21st

1 For every positive integer n we define a_n as the last digit of the sum $1+2+\cdots+n$. Compute $a_1+a_2+\cdots+a_{1992}$.

2 Given the positive real numbers $a_1<a_2<\cdots<a_n$, consider the function

$$f(x)=\frac{a_1}{x+a_1}+\frac{a_2}{x+a_2}+\cdots+\frac{a_n}{x+a_n}$$

Determine the sum of the lengths of the disjoint intervals formed by all the values of x such that $f(x)>1$.

3 Let ABC be an equilateral triangle of sidelength 2 and let ω be its incircle.

a) Show that for every point P on ω the sum of the squares of its distances to A, B, C is 5.

b) Show that for every point P on ω it is possible to construct a triangle of sidelengths AP, BP, CP. Also, the area of such triangle is $\frac{\sqrt{3}}{4}$.

Day 2 September 22nd

1 Let $\{a_n\}_{n\geq 0}$ and $\{b_n\}_{n\geq 0}$ be two sequences of integer numbers such that:

i. $a_0=0$, $b_0=8$.

ii. For every $n\geq 0$, $a_{n+2}=2a_{n+1}-a_n+2$, $b_{n+2}=2b_{n+1}-b_n$.

iii. $a_n^2+b_n^2$ is a perfect square for every $n\geq 0$.

Find at least two values of the pair $(a_{1992},\, b_{1992})$.

2 Given a circle Γ and the positive numbers h and m, construct with straight edge and compass a trapezoid inscribed in Γ, such that it has altitude h and the sum of its parallel sides is m.

3 In a triangle ABC, points A_1 and A_2 are chosen in the prolongations beyond A of segments AB and AC, such that $AA_1=AA_2=BC$. Define analogously points B_1, B_2, C_1, C_2. If $[ABC]$ denotes the area of triangle ABC, show that $[A_1A_2B_1B_2C_1C_2]\geq 13[ABC]$.

