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Day 1

1	A number is called *capicua* if when it is written in decimal notation, it can be read equal from left to right as from right to left; for example: $8, 23432, 6446$. Let $x_1 < x_2 < \cdots < x_i < x_{i+1}, \cdots$ be the sequence of all capicua numbers. For each i define $y_i = x_{i+1} - x_i$. How many distinct primes contains the set $\{y_1, y_2, \ldots\}$?
2	Show that for every convex polygon whose area is less than or equal to 1, there exists a parallelogram with area 2 containing the polygon.
3	Let $\mathbb{N}^* = \{1, 2, \ldots\}$. Find al the functions $f : \mathbb{N}^* \to \mathbb{N}^*$ such that: (1) If $x < y$ then $f(x) < f(y)$. (2) $f(yf(x)) = x^2 f(xy)$ for all $x, y \in \mathbb{N}^*$.

Day 2

1	Let ABC be an equilateral triangle and Γ its incircle. If D and E are points on the segments AB and AC such that DE is tangent to Γ, show that $\frac{AD}{DB} + \frac{AE}{EC} = 1$.
2	Let P and Q be two distinct points in the plane. Let us denote by $m(PQ)$ the segment bisector of PQ. Let S be a finite subset of the plane, with more than one element, that satisfies the following properties: (i) If P and Q are in S, then $m(PQ)$ intersects S. (ii) If P_1Q_1, P_2Q_2, P_3Q_3 are three diferent segments such that its endpoints are points of S, then, there is non point in S such that it intersects the three lines $m(P_1Q_1)$, $m(P_2Q_2)$, and $m(P_3Q_3)$. Find the number of points that S may contain.
3	Two nonnegative integers a and b are *tuanis* if the decimal expression of $a+b$ contains only 0 and 1 as digits. Let A and B be two infinite sets of non negative integers such that B is the set of all the *tuanis* numbers to elements of the set A and A the set of all the *tuanis* numbers to elements of the set B. Show that in at least one of the sets A and B there is an infinite number of pairs (x, y) such that $x - y = 1$.