IberoAmerican 1998

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Day 1

1 Given 98 points in a circle. Mary and Joseph play alternatively in the next way:

- Each one draw a segment joining two points that have not been joined before.

The game ends when the 98 points have been used as end points of a segments at least once. The winner is the person that draw the last segment. If Joseph starts the game, who can assure that is going to win the game.

2 The circumference inscribed on the triangle ABC is tangent to the sides BC, CA and AB on the points D, E and F, respectively. AD intersect the circumference on the point Q. Show that the line EQ meet the segment AF at its midpoint if and only if $AC = BC$.

3 Find the minimum natural number n with the following property: between any collection of n distinct natural numbers in the set $\{1, 2, \dots, 999\}$ it is possible to choose four different $a,\ b,\ c,\ d$ such that: $a + 2b + 3c = d$.

Day 2

1 There are representants from n different countries sit around a circular table ($n \geq 2$), in such way that if two representants are from the same country, then, their neighbors to the right are not from the same country. Find, for every n, the maximal number of people that can be sit around the table.

2 Find the maximal possible value of n such that there exist points $P_1, P_2, P_3, \dots, P_n$ in the plane and real numbers $r_1, r_2, \dots, r_n$ such that the distance between any two different points P_i and P_j is $r_i + r_j$.

3 Let λ the positive root of the equation $t^2 - 1998t - 1 = 0$. It is defined the sequence $x_0, x_1, x_2, \dots, x_n, \dots$ by $x_0 = 1,\ x_{n+1} = \lfloor \lambda x_n \rfloor$ for $n = 1, 2 \dots$ Find the remainder of the division of x_{1998} by 1998.

Note: $\lfloor x \rfloor$ is the greatest integer less than or equal to x.