IberoAmerican 2000

www.artofproblemsolving.com/community/c4540

by carlosbr

Day 1

---

1 A regular polygon of n sides ($n \geq 3$) has its vertex numbered from 1 to n. One draws all the diagonals of the polygon. Show that if n is odd, it is possible to assign to each side and to each diagonal an integer number between 1 and n, such that the next two conditions are simultaneously satisfied:

(a) The number assigned to each side or diagonal is different to the number assigned to any of the vertices that is endpoint of it.

(b) For each vertex, all the sides and diagonals that have it as an endpoint, have different number assigned.

---

2 Let S_1 and S_2 be two circumferences, with centers O_1 and O_2 respectively, and secants on M and N. The line t is the common tangent to S_1 and S_2 closer to M. The points A and B are the intersection points of t with S_1 and S_2, C is the point such that BC is a diameter of S_2, and D the intersection point of the line O_1O_2 with the perpendicular line to AM through B. Show that M, D and C are collinear.

---

3 Find all the solutions of the equation

$$(x+1)^y - x^z = 1$$

For x, y, z integers greater than 1.

Day 2

---

1 From an infinite arithmetic progression $1, a_1, a_2, \dots$ of real numbers some terms are deleted, obtaining an infinite geometric progression $1, b_1, b_2, \dots$ whose ratio is q. Find all the possible values of q.

---

2 There are a buch of 2000 stones. Two players play alternatively, following the next rules:

(a)On each turn, the player can take 1, 2, 3, 4 or 5 stones **of** the bunch.
(b) On each turn, the player has forbidden to take the exact same amount of stones that the other player took just before of him in the last play.

The loser is the player who can't make a valid play. Determine which player has winning strategy and give such strategy.

3 A convex hexagon is called *pretty* if it has four diagonals of length 1, such that their endpoints are all the vertex of the hexagon.
(a) Given any real number k with $0 < k < 1$ find a *pretty* hexagon with area equal to k
(b) Show that the area of any *pretty* hexagon is less than 1.