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Day 1

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1 We say that a natural number n is *charrua* if it satisfy simultaneously the following conditions:
- Every digit of n is greater than 1.
- Every time that four digits of n are multiplied, it is obtained a divisor of n

Show that every natural number k there exists a *charrua* number with more than k digits.

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2 The incircle of the triangle $\triangle ABC$ has center at O and it is tangent to the sides BC, AC and AB at the points X, Y and Z, respectively. The lines BO and CO intersect the line YZ at the points P and Q, respectively.

Show that if the segments XP and XQ has the same length, then the triangle $\triangle ABC$ is isosceles.

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3 Let S be a set of n elements and $S_1,\ S_2,\dots,\ S_k$ are subsets of S ($k\geq 2$), such that every one of them has at least r elements.

Show that there exists i and j, with $1\leq i<j\leq k$, such that the number of common elements of S_i and S_j is greater or equal to: $r-\frac{nk}{4(k-1)}$

Day 2

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1 Find the maximum number of increasing arithmetic progressions that can have a finite sequence of real numbers $a_1<a_2<\cdots<a_n$ of $n\geq 3$ real numbers.

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2 In a board of 2000×2001 squares with integer coordinates (x,y), $0\leq x\leq 1999$ and $0\leq y\leq 2000$. A ship in the table moves in the following way: before a move, the ship is in position (x,y) and has a velocity of (h,v) where x,y,h,v are integers. The ship chooses new velocity (h',v') such that $h'-h,v'-v\in\{-1,0,1\}$. The new position of the ship will be (x',y') where x' is the remainder of the division of $x+h'$ by 2000 and y' is the remainder of the division of $y+v'$ by $2001.$

There are two ships on the board: The Martian ship and the Human trying to capture it. Initially each ship is in a different square and has velocity $(0,0)$. The Human is the first to move; thereafter they continue moving alternatively.

Is there a strategy for the Human to capture the Martian, independent of the initial positions and the Martians moves?

Note: The Human catches the Martian ship by reaching the same position as the Martian ship after the same move.

3 Show that it is impossible to cover a unit square with five equal squares with side $s < \frac{1}{2}$.