**IberoAmerican 2007**

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**Day 1** September 11th

**1** Given an integer $m$, define the sequence $\{a_n\}$ as follows:

$$a_1 = \frac{m}{2},\ a_{n+1} = a_n \lceil a_n \rceil,\ \text{if } n \geq 1$$

Find all values of $m$ for which $a_{2007}$ is the first integer appearing in the sequence.

Note: For a real number $x$, $\lceil x \rceil$ is defined as the smallest integer greater or equal to $x$. For example, $\lceil \pi \rceil = 4$, $\lceil 2007 \rceil = 2007$.

**2** Let $ABC$ be a triangle with incenter $I$ and let $\Gamma$ be a circle centered at $I$, whose radius is greater than the inradius and does not pass through any vertex. Let $X_1$ be the intersection point of $\Gamma$ and line $AB$, closer to $B$; $X_2$, $X_3$ the points of intersection of $\Gamma$ and line $BC$, with $X_2$ closer to $B$; and let $X_4$ be the point of intersection of $\Gamma$ with line $CA$ closer to $C$. Let $K$ be the intersection point of lines $X_1X_2$ and $X_3X_4$. Prove that $AK$ bisects segment $X_2X_3$.

**3** Two teams, $A$ and $B$, fight for a territory limited by a circumference.
$A$ has $n$ blue flags and $B$ has $n$ white flags ($n \geq 2$, fixed). They play alternatively and $A$ begins the game. Each team, in its turn, places one of his flags in a point of the circumference that has not been used in a previous play. Each flag, once placed, cannot be moved.

Once all $2n$ flags have been placed, territory is divided between the two teams. A point of the territory belongs to $A$ if the closest flag to it is blue, and it belongs to $B$ if the closest flag to it is white. If the closest blue flag to a point is at the same distance than the closest white flag to that point, the point is neutral (not from $A$ nor from $B$). A team wins the game is their points cover a greater area that that covered by the points of the other team. There is a draw if both cover equal areas.
Prove that, for every $n$, team $B$ has a winning strategy.

**Day 2** September 12th

**4** In a $19 \times 19$ board, a piece called *dragon* moves as follows: It travels by four squares (either horizontally or vertically) and then it moves one square more in a direction perpendicular to its previous direction. It is known that, moving so, a dragon can reach every square of the board.

The *draconian distance* between two squares is defined as the least number of moves a dragon needs to move from one square to the other.

Let $C$ be a corner square, and $V$ the square neighbor of $C$ that has only a point in common with $C$. Show that there exists a square $X$ of the board, such that the draconian distance between $C$ and $X$ is greater than the draconian distance between $C$ and $V$.

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**5** Let's say a positive integer $n$ is *atresvido* if the set of its divisors (including 1 and $n$) can be split in in 3 subsets such that the sum of the elements of each is the same. Determine the least number of divisors an atresvido number can have.

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**6** Let $\mathcal{F}$ be a family of hexagons $H$ satisfying the following properties:

i) $H$ has parallel opposite sides.

ii) Any 3 vertices of $H$ can be covered with a strip of width 1.

Determine the least $\ell \in \mathbb{R}$ such that every hexagon belonging to $\mathcal{F}$ can be covered with a strip of width $\ell$.

Note: A strip is the area bounded by two parallel lines separated by a distance $\ell$. The lines belong to the strip, too.

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