IberoAmerican 2010

www.artofproblemsolving.com/community/c4550

by Concyclicboy

Day 1

1 There are ten coins a line, which are indistinguishable. It is known that two of them are false and have consecutive positions on the line. For each set of positions, you may ask how many false coins it contains. Is it possible to identify the false coins by making only two of those questions, without knowing the answer to the first question before making the second?

2 Determine if there are positive integers a, b such that all terms of the sequence defined by

$$x_1 = 2010, x_2 = 2011 x_{n+2} = x_n + x_{n+1} + a\sqrt{x_n x_{n+1} + b} \quad (n \geq 1)$$

are integers.

3 The circle Γ is inscribed to the scalene triangle ABC. Γ is tangent to the sides BC, CA and AB at D, E and F respectively. The line EF intersects the line BC at G. The circle of diameter GD intersects Γ in R ($R \neq D$). Let P, Q ($P \neq R, Q \neq R$) be the intersections of Γ with BR and CR, respectively. The lines BQ and CP intersects at X. The circumcircle of CDE meets QR at M, and the circumcircle of BDF meet PR at N. Prove that PM, QN and RX are concurrent.

Author: Arnoldo Aguilar, El Salvador

Day 2

1 The arithmetic, geometric and harmonic mean of two distinct positive integers are different numbers. Find the smallest possible value for the arithmetic mean.

2 Let $ABCD$ be a cyclic quadrilateral whose diagonals AC and BD are perpendicular. Let O be the circumcenter of $ABCD$, K the intersection of the diagonals, $L \neq O$ the intersection of the circles circumscribed to OAC and OBD, and G the intersection of the diagonals of the quadrilateral whose vertices are the midpoints of the sides of $ABCD$. Prove that O, K, L and G are collinear

3 Around a circular table sit 12 people, and on the table there are 28 vases. Two people can see each other, if and only if there is no vase lined with them. Prove that there are at least two people who can be seen.