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Art of Problem Solving

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Day 1

1 A set S of positive integers is said to be *channeler* if for any three distinct numbers $a,b,c \in S$, we have $a \mid bc$, $b \mid ca$, $c \mid ab$.

a) Prove that for any finite set of positive integers $\{c_1, c_2, \ldots, c_n\}$ there exist infinitely many positive integers k, such that the set $\{kc_1, kc_2, \ldots, kc_n\}$ is a channeler set.

b) Prove that for any integer $n \geq 3$ there is a channeler set who has exactly n elements, and such that no integer greater than 1 divides all of its elements.

2 Let X and Y be the diameter's extremes of a circunference Γ and N be the midpoint of one of the arcs XY of Γ. Let A and B be two points on the segment XY. The lines NA and NB cuts Γ again in C and D, respectively. The tangents to Γ at C and at D meets in P. Let M the the intersection point between XY and NP. Prove that M is the midpoint of the segment AB.

3 Let $A = \{1, ..., n\}$ with $n{>}5$. Prove that one can find B a finite set of positive integers such that A is a subset of B and

$$\sum_{x \in B} x^2 = \prod_{x \in B} x$$

Day 2

4 Let Γ be a circunference and O its center. AE is a diameter of Γ and B the midpoint of one of the arcs AE of Γ. The point $D \neq E$ in on the segment OE. The point C is such that the quadrilateral $ABCD$ is a parallelogram, with AB parallel to CD and BC parallel to AD. The lines EB and CD meets at point F. The line OF cuts the minor arc EB of Γ at I.

Prove that the line EI is the angle bissector of $\angle BEC$.

5 Let A and B be two sets such that $A \cup B$ is the set of the positive integers, and $A \cap B$ is the empty set. It is known that if two positive integers have a prime larger than 2013 as their difference, then one of them is in A and the other is in B. Find all the possibilities for the sets A and B.

6 A *beautiful configuration* of points is a set of n colored points, such that if a triangle with vertices in the set has an angle of at least 120 degrees, then exactly 2 of its vertices are colored with the same color. Determine the maximum possible value of n.