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Day 1 July 8th

1 Suppose that each of the 5 persons knows a piece of information, each piece is different, about a certain event. Each time person A calls person B, A gives B all the information that A knows at that moment about the event, while B does not say to A anything that he knew.

(a) What is the minimum number of calls are necessary so that everyone knows about the event?
(b) How many calls are necessary if there were n persons?

2 Find a positive integer n with 1000 digits, all distinct from zero, with the following property: it's possible to group the digits of n into 500 pairs in such a way that if the two digits of each pair are multiplied and then add the 500 products, it results a number m that is a divisor of n.

3 The digits of a calculator (with the exception of 0) are shown in the form indicated by the figure below, where there is also a button "+":
6965
Two players A and B play in the following manner. A turns on the calculator and presses a digit, and then presses the button "+". A passes the calculator to B, which presses a digit in the same row or column with the one pressed by A that is not the same as the last one pressed by A; and then presses + and returns the calculator to A, repeating the operation in this manner successively. The first player that reaches or exceeds the sum of 31 loses the game. Which of the two players have a winning strategy and what is it?

Day 2 July 9th

4 In the trapezoid $ABCD$ with bases AB and CD, let M be the midpoint of side DA. If $BC = a$, $MC = b$ and $\angle MCB = 150^\circ$, what is the area of trapezoid $ABCD$ as a function of a and b?

5 Let a be an odd positive integer greater than 17 such that $3a-2$ is a perfect square. Show that there exist distinct positive integers b and c such that $a+b, a+c, b+c$ and $a+b+c$ are four perfect squares.

6 Denote S as the subset of $\{1, 2, 3, \dots, 1000\}$ with the property that none of the sums of two different elements in S is in S. Find the maximum number of elements in S.