CentroAmerican 2000

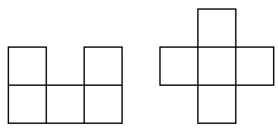
www.artofproblemsolving.com/community/c4556

by Jutaro

Day 1 July 11th

1 Find all three-digit numbers abc (with $a \neq 0$) such that $a^2 + b^2 + c^2$ is a divisor of 26.

2 Determine all positive integers n such that it is possible to tile a $15 \times n$ board with pieces shaped like this:

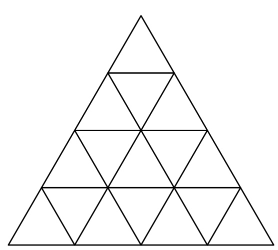


3 Let $ABCDE$ be a convex pentagon. If P, Q, R and S are the respective centroids of the triangles ABE, BCE, CDE and DAE, show that $PQRS$ is a parallelogram and its area is $2/9$ of that of $ABCD$.

Day 2 July 12th

1 Write an integer on each of the 16 small triangles in such a way that every number having at least two neighbors is equal to the difference of two of its neighbors.

Note: Two triangles are said to be neighbors if they have a common side.



2 Let ABC be an acute-angled triangle. C_1 and C_2 are two circles of diameters AB and AC, respectively. C_2 and AB intersect again at F, and C_1 and AC intersect again at E. Also, BE

meets C_2 at P and CF meets C_1 at Q. Prove that $AP = AQ$.

---

3 Let's say we have a *nice* representation of the positive integer n if we write it as a sum of powers of 2 in such a way that there are at most two equal powers in the sum (representations differing only in the order of their summands are considered to be the same).

a) Write down the 5 nice representations of 10.

b) Find all positive integers with an even number of nice representations.
